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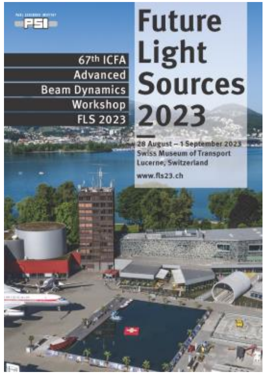

Scaling of Beam Collective Effects with Bunch Charge in the CompactLight FEL

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on behalf of the CompactLight Collaboration

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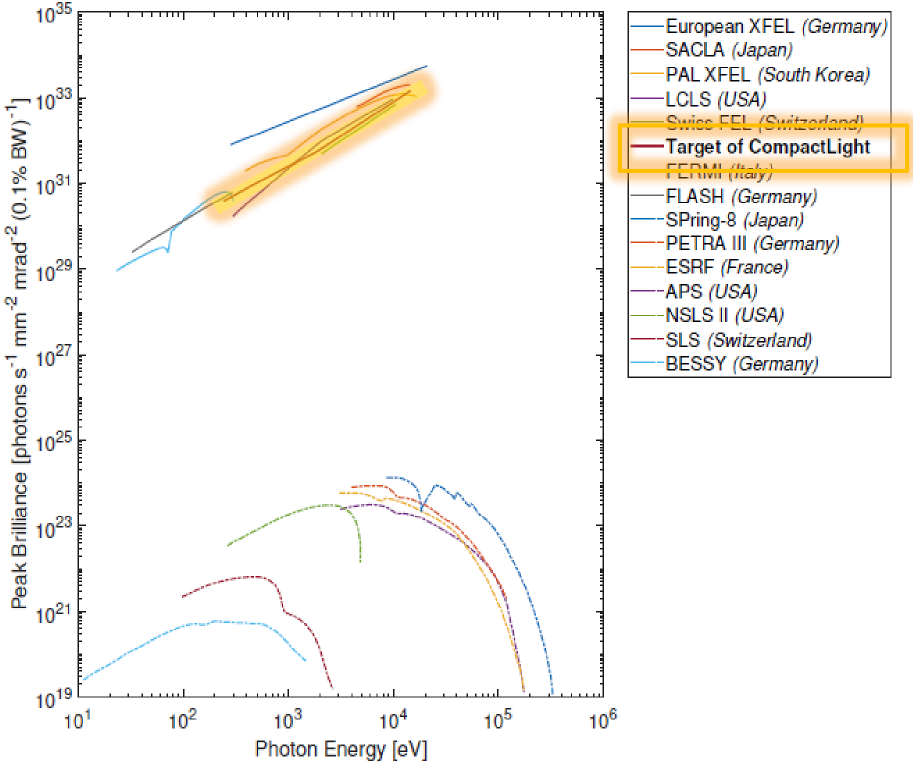
Introduction



$\lambda_R \sim$ 1 Å (16 keV)

λ_u = 10 mm, K ~ 1 (SCU+AB)

$$E_e = m_e c^2 \sqrt{\frac{\lambda_u}{2\lambda_R}\left(1+\frac{K^2}{2}\right)} = 5.5\ \text{GeV}$$

1 kHz NC, 65 MV/m

$$L_{linac} \approx \frac{E_e}{G} = 100\ \text{m active length}$$

$$\boldsymbol{I \approx \frac{75pC}{\sqrt{2\pi}\,30fs} = 1kA, \qquad \rho \approx 0.01\%}$$

$$E_{sase} \approx 2.4 \times \sigma_{t,b} \times \rho I E < \boldsymbol{100\mu J}$$

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Motivation

- ☐ Scientific cases for several 100's μJ in soft X-rays
  - ➢ Keep ***same final peak current*** to keep L_{sat} and P_{sat} almost fixed
    - *0.4 kA @ 1 kHz (Soft-X), 4.5 kA @ 0.1 kHz (Hard-X)*
  - ➢ Bunch duration is increased proportionally to the bunch charge
  - ➢ We expect $E_{sase} \propto Q$
- ☐ *But:* collective effects are also $\propto Q_b$, enlarging the projected e-beam emittances
  - ➢ In reality $E_{sase} \propto Q^{\nu},\ \nu < 1$
- ☐ Goal: set up an analytical model to estimate E_{sase} and F_{coh} vs. Q (< 1 nC)

Space Charge-dominated Emittance

[1] J. Rosenzweig and E. Colby, TESLA note 95-04

❑ Envelope eq., cylindrical beam, $\varepsilon_{th} \rightarrow 0$:

$$\sigma_x'' + \sigma_x' \frac{(\beta\gamma)'}{\beta\gamma} + K\sigma_x = \kappa_{sc}\sigma_x$$

with $\kappa_{sc} := \frac{2I}{I_A}\frac{1}{(\beta\gamma)^3\sigma_x^2} f\left(\frac{\sigma_x}{\beta\gamma\sigma_z}\right) = \left(\frac{2c}{I_A}\frac{f/g}{\beta^2\gamma^3}\right)\frac{Q}{\sigma_z\sigma_x^2} \equiv const.$ for increasing Q

- *Keep the aspect ratio **f** and the longitudinal profile **g** constant* $\Longrightarrow$ $\frac{Q}{\sigma_z\sigma_x^2} \equiv const.$

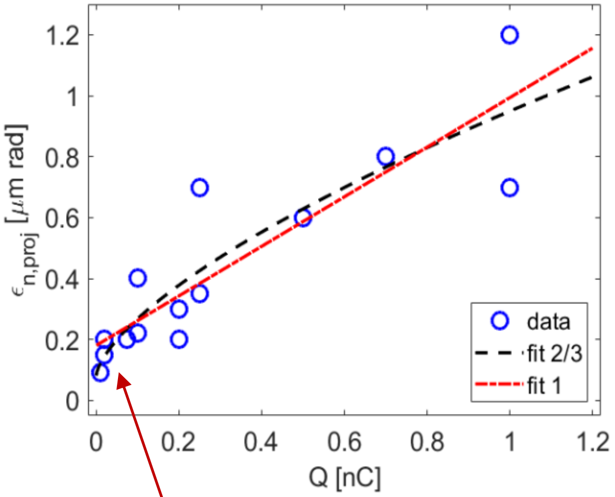
❑ For linear SC (blow-out), $\sigma_i \propto Q^{1/3}$. Since $\sigma_x = \sqrt{\varepsilon_x\beta_x}$ and $\beta_x \approx const.$, we find:

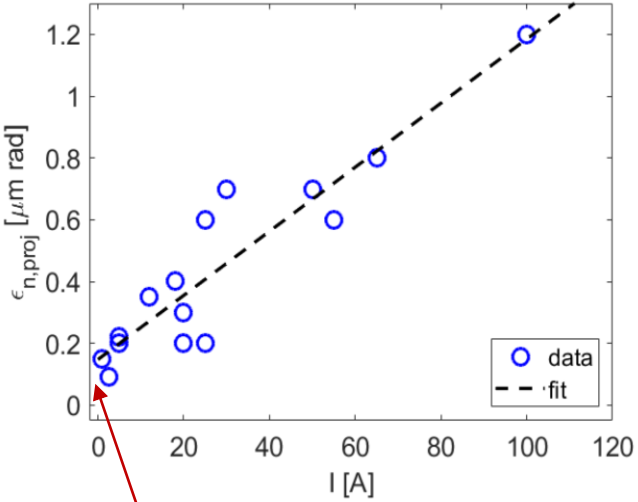
$\varepsilon_x[\mu m] \approx a \times Q[nC]^{2/3}$ and $\varepsilon_x[\mu m] \approx b \times I[kA]$

❑ If we force $\sigma_z = const.$ through laser shaping, we find instead: $\varepsilon_x[\mu m] \approx c \times Q[nC]$

Fitting Experimental Data

Heterogeneous ensemble of experimental data from RF-PI (2009–2018, <1nC):





$\varepsilon_x \sim Q^{2/3}$ is able to capture emittances of very low charges

The model has an offset for $I \rightarrow 0$, which takes in to account the thermal emittance

$$\varepsilon_x[\mu m] \approx Q[nC]^{2/3}$$

agrees with simulations in [1] for SC-dominated beams.

Take simul. of 75pC as a ref. (“0”):

$$\sigma_{z,i} = \left(\frac{Q}{Q_0}\right)^{\frac{1}{3}} \sigma_{z,0}$$

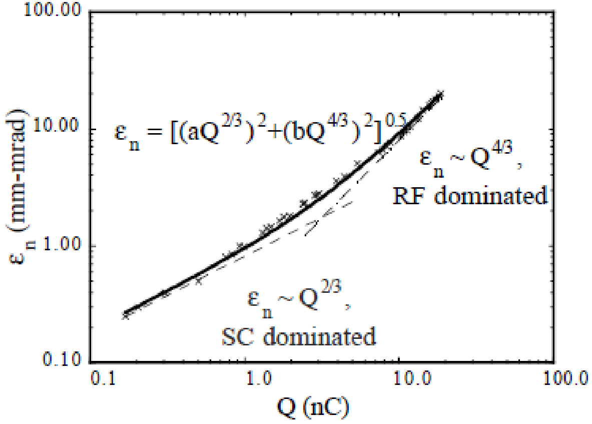
$$\Rightarrow \quad I_i = I_0 \left(\frac{Q}{Q_0}\right)^{\frac{2}{3}}$$

$$CF = \frac{I}{I_i} = \frac{I}{I_0}\left(\frac{Q_0}{Q}\right)^{2/3}$$

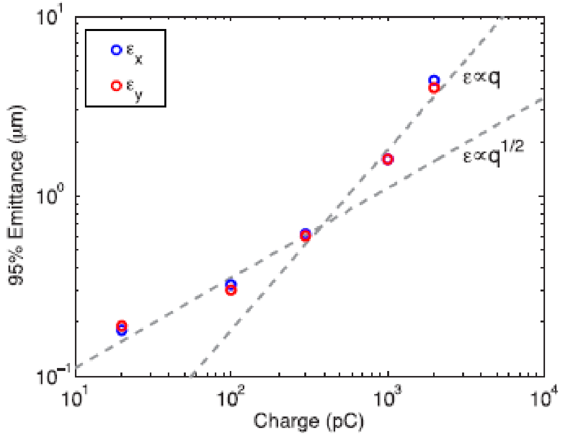
$$\sigma_{z,f} = \sigma_{z,i} / CF$$

Examples from the literature

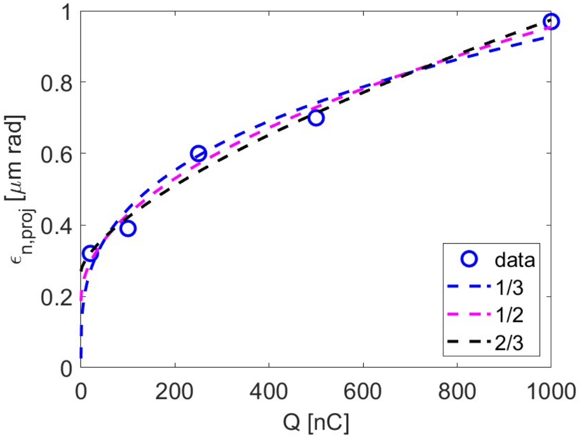
AJ. Rosenzweig and E. Colby, IEEE 1996
Parmela simulations of NC RF optimized Gun



A. Bartnik et al., PRst_AB 18, 083401 (2015)
Cornell DC Gun



Data courtesy of Y. Chen for EU-XFEL at FLS'23



Most authors agree on the fact that, in SC-dominated regime, the emittance scales like $\varepsilon_{x,y} \propto Q^{\nu}$, with $\nu = \left[\frac{1}{2}, \frac{2}{3}\right]$. For Q > 100s pC, one can observe $\nu \rightarrow 1$.

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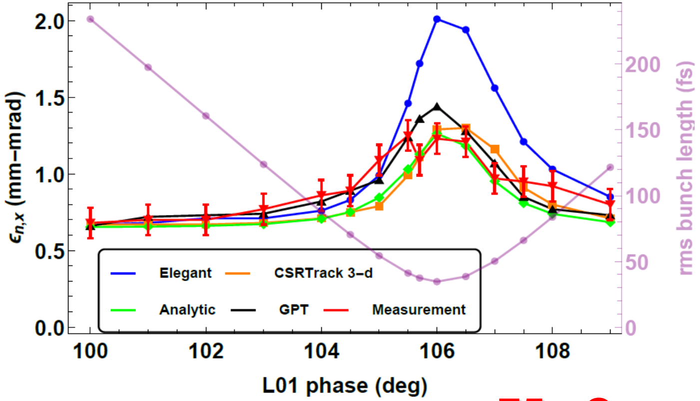
Coherent Synchrotron Radiation



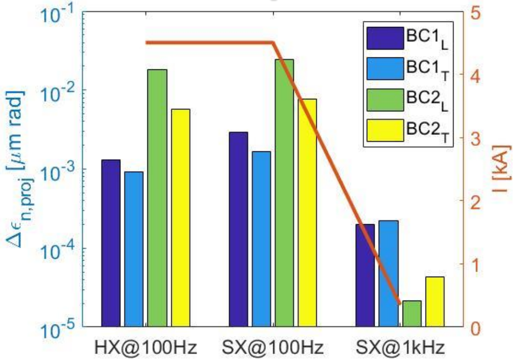
$$\Delta\varepsilon_{n,L} = 7.5\times10^{-3}\frac{\beta}{\gamma}\left(\frac{N r_e L_b^2}{R^{\frac{5}{3}}\sigma_z^{\frac{4}{3}}}\right)^2$$

$$\Delta\varepsilon_{n,T} = 2.5\times10^{-2}\frac{\beta}{\gamma}\left(\frac{N r_e L_b}{R\sigma_z}\right)^2$$

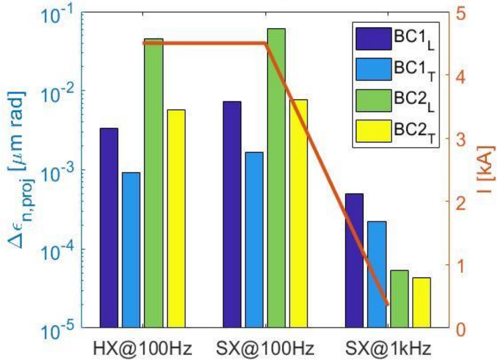
A. Brynes et al., NJP 20, 073035 (2018)
G. Stupakov, *arXiv*, 1901:10745 (2019)

The model agrees with 3-D codes. Longitudinal CSR field dominates.

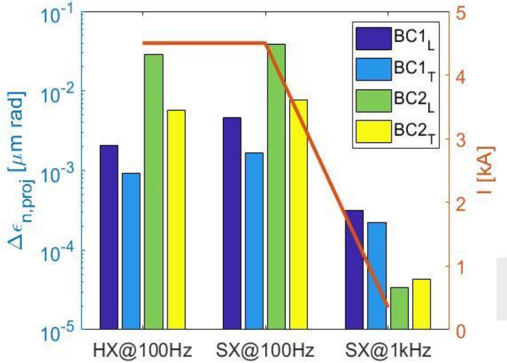
75 pC



150 pC



300 pC



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Single Bunch Beam Break Up

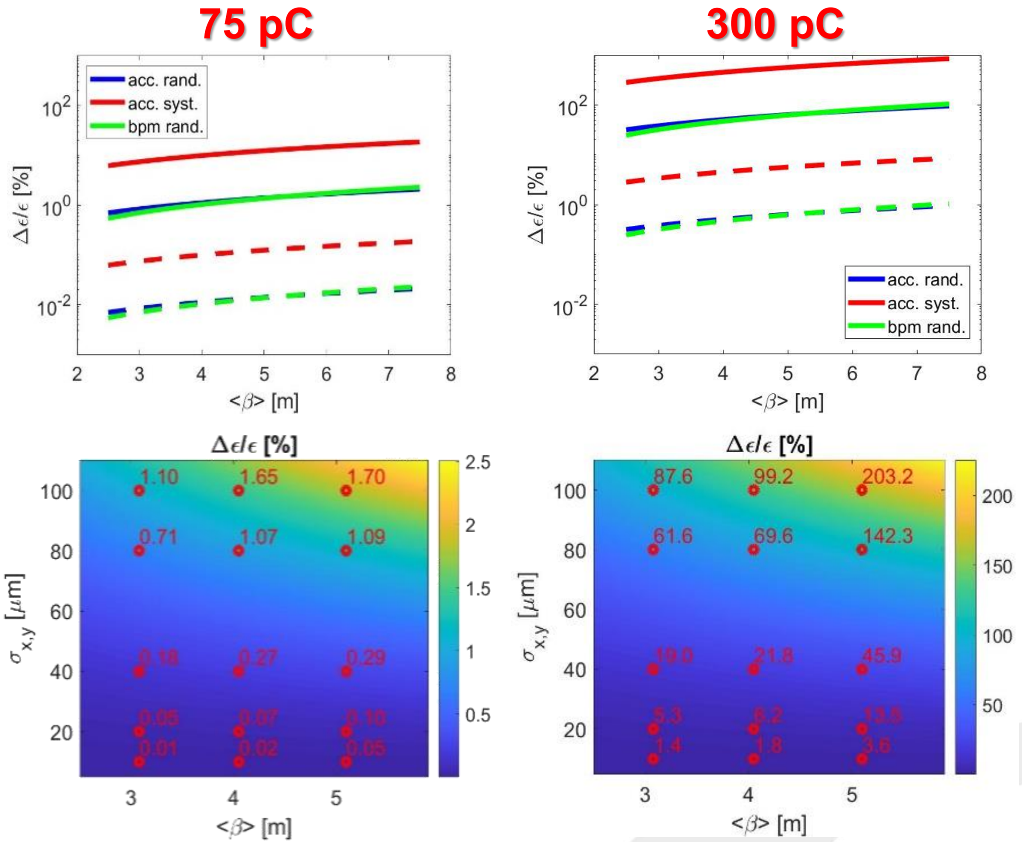
$$\Delta\varepsilon_{n,w_T} \propto \Delta^2 \times \mathrm{N_e^2 w_T^2}(2\sigma_z)\frac{\mathrm{L_{rf}}}{\alpha G_{rf}}F(\Delta\mu)\left[\left(\frac{\gamma_f}{\gamma_i}\right)^{\alpha}-1\right]$$

with $\overline{\beta_u} \propto \gamma^{\alpha}, \alpha < 1$

1. RF random misalignment
2. RF systematic misalignment (2-by-2)
3. BPMs misalignment (FODO)

T. Raubenheimer, PRST-AB 3, 121002 (2000)

The model agrees well with PLACET simulations. Still valid at large emittance growths.



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Peak Brilliance: *3-D "slice" corrections*

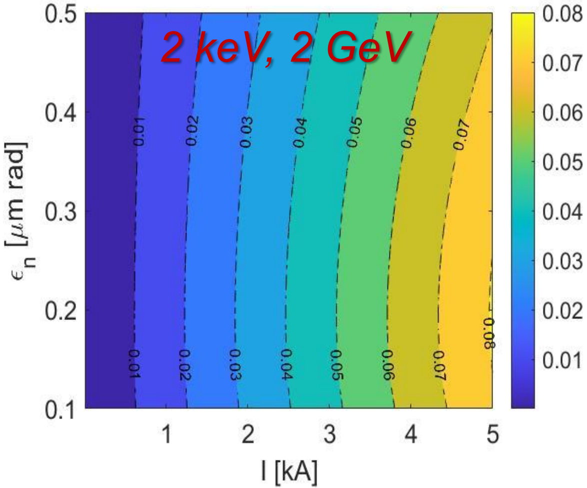
M. Xie, Proc. of PAC'95
G. Dattoli and M. Renieri, Laser Handbook (1986)
E. Saldin et al., in FEL'04 and NJP 12 (2010)

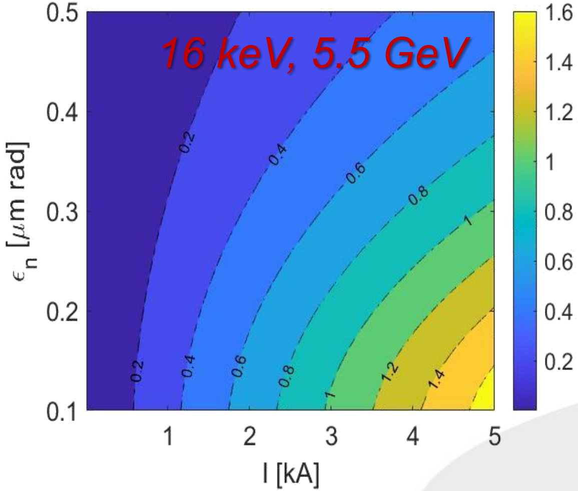
$$B_{ph} \cong 4.5 \times 10^{30} \frac{I(kA) \times E(GeV)}{\lambda(nm)} \times \delta$$

in $[\#\mathrm{ph}/s/\mathrm{mm}^2\,/mrad^2/0.1\,\%\mathrm{bw}]$

- Fraction of unity
- Related to the coherent fraction of light
- Depends on current and slice emittances through $L_{G,3D}$

Sensitivity study, no functional dependence yet





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FEL Pulse Energy: *3-D “projected” corrections*

T. Tanaka et al., NIM A 528 (2004) 172
S. Di Mitri & S. Spampinati, PRST-AB 17, (2014)

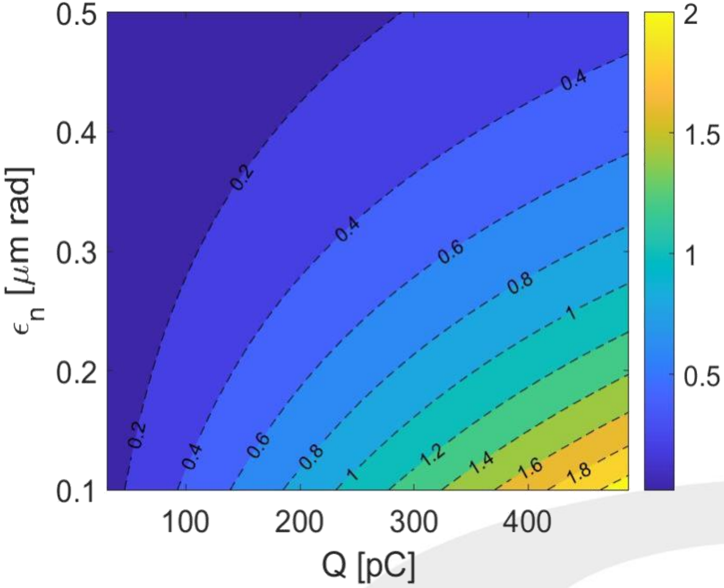
$$E_{sase} \cong 0.6 \times \mathrm{P_{sat}} \times \sqrt{2\pi}\sigma_t \cong 2.4\rho I E \frac{Q}{I} = 2.4 E Q \rho$$

from spiky emission in t-domain from a flat-top bunch

$$\rho = \frac{\rho_{3D}}{1+\kappa\langle\theta_{coll}^2\rangle} \approx \frac{\rho_{3D}}{1+\left(\frac{L_{g,3D}}{2\pi\bar{\beta}_u}\right)\frac{\Delta\varepsilon_{n,pr}}{\varepsilon_{n,sl}}}$$

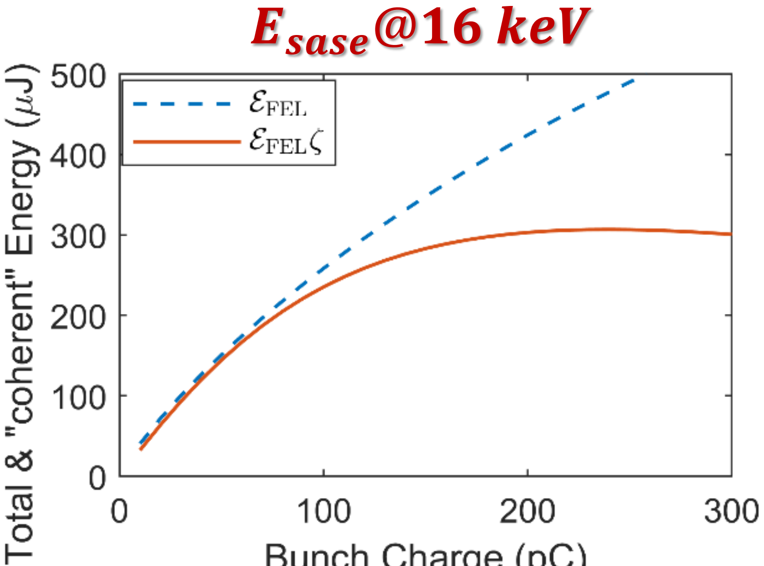
Projected emittance growth reduces the overlap of photons and electrons during the exp amplification

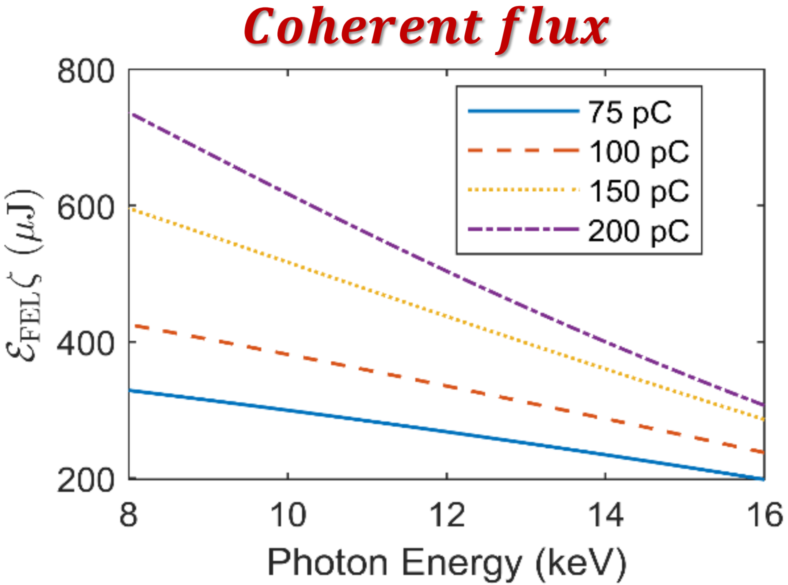
E_{sase} [μJ], @16 keV, I=4.5 kA



FEL Pulse Energy: *Q-scaling*

Finally, we apply the scaling $\varepsilon_{n,sl}= \varepsilon_{n,sl}(Q)$, include $\Delta\varepsilon_{n,pr}(Q)$ from CSR and BBU, and estimate the coherent fraction of SASE flux, $\zeta = \frac{1.1\epsilon^{1/4}}{1+0.15\epsilon^{9/4}}$, with $\epsilon := 2\pi\varepsilon_x/\lambda$.





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Conclusions

- ✓ The model of “invariant beam envelope” predicts space charge-emittance reasonably well for a large variety of optimized PI.
- ✓ A strategy to estimate $E_{sase}(\mathrm{Q})$ is presented. 3-D slice and projected corrections to the brilliance are included in a semi-analytical, self-consistent model.
- ✓ The model is expected to highlight dominant FEL dependences from “macroscopic” e-beam parameters.

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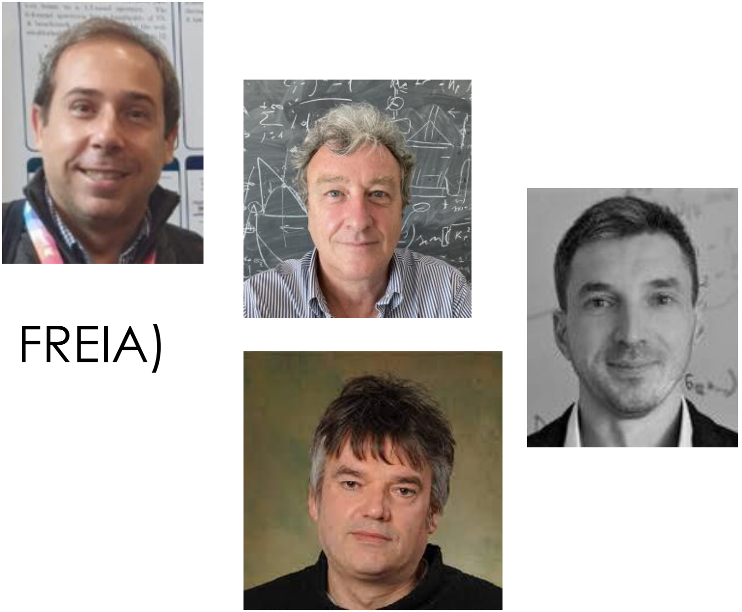
Acknowledgements



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Thank You for Your attention

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