An Efficient Optimization of a Burst Mode-operated Fabry Perot Cavity for Compton Light sources

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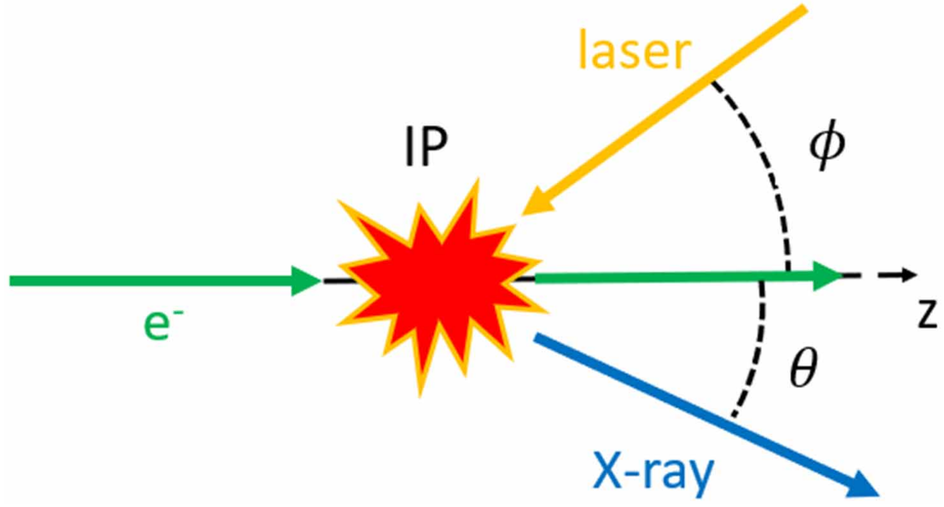
29/08/2023

Contents

Inverse Compton Scattering

= The scattering of a low energy photon from an EM field to a high-energy photon (X-ray or gamma ray) during the interaction with a charged particle.

One of the few compact light sources capable of generating MeV photons with a small energy bandwidth.



$$N_\gamma = \sigma_c \frac{N_e N_{\text{laser}} \cos(\phi/2)}{2\pi \sigma_{\gamma,y} \sqrt{\sigma_{\gamma,x}^2 \cos^2(\phi/2) + \sigma_{\gamma,z}^2 \sin^2(\phi/2)}}$$

Total flux

$$\mathcal{B} = \frac{\mathcal{F}}{4\pi^2 \sigma_{\gamma,x} \sqrt{\epsilon_x/\beta_x} \sigma_{\gamma,y} \sqrt{\epsilon_y/\beta_y}}$$

Average brilliance

$$\frac{\sigma_{E_\gamma}}{E_\gamma} = \sqrt{\left(\frac{\sigma_{E_\theta}}{E_\theta}\right)^2 + \left(2\frac{\sigma_{E_e}}{E_e}\right)^2 + \left(\frac{\sigma_{E_{\text{laser}}}}{E_{\text{laser}}}\right)^2 + \left(\frac{\sigma_{E_\epsilon}}{E_\epsilon}\right)^2}$$

Photon bandwidth

$$E_{\text{X-ray}} = 2\gamma^2 E_{\text{laser}} \frac{1 + \cos\phi}{1 + \gamma^2\theta^2}$$

Photon energy

How to maximise the photon flux?

~ Laser effective energy

Crossing angle $(2° - 5°)$

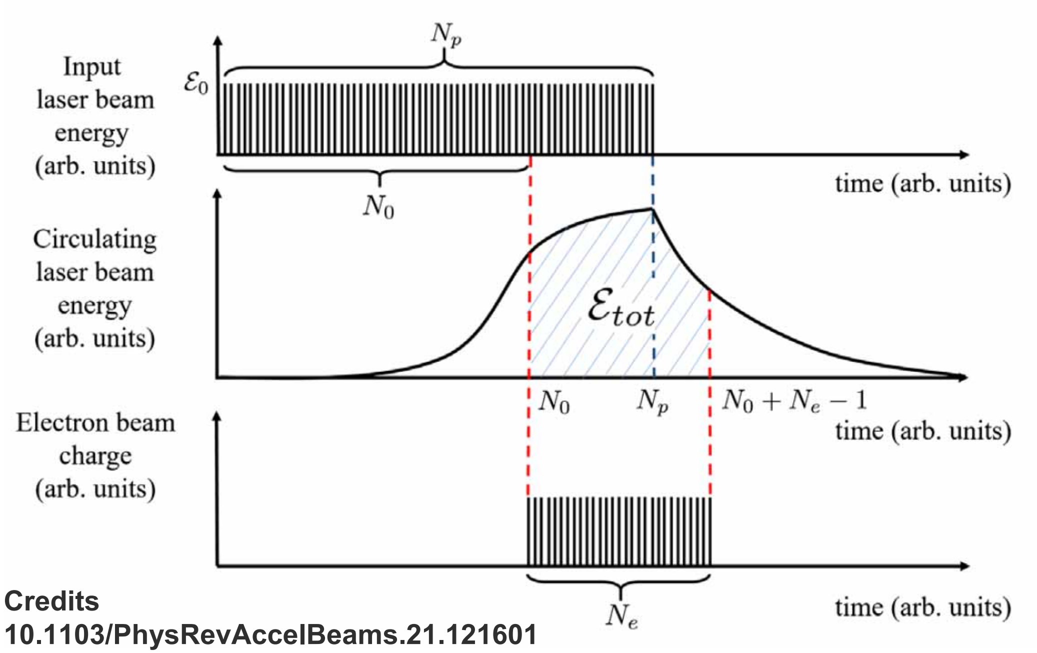
~ Train charge

$$N_\gamma = \sigma_c \frac{N_e N_{laser} \cos(\phi/2)}{2\pi\sigma_{\gamma,y}\sqrt{\sigma_{\gamma,x}^2 \cos^2(\phi/2) + \sigma_{\gamma,z}^2 \sin^2(\phi/2)}}$$

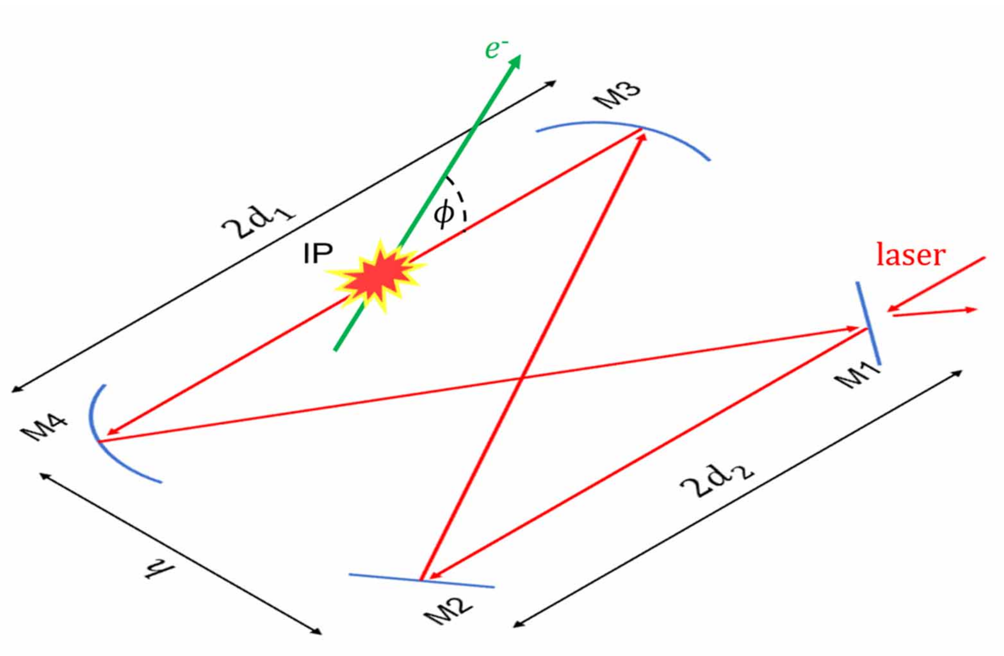
Source size

$$\mathcal{F} \approx N_\gamma f_{eff}$$

Why burst mode operation?



Credits
10.1103/PhysRevAccelBeams.21.121601

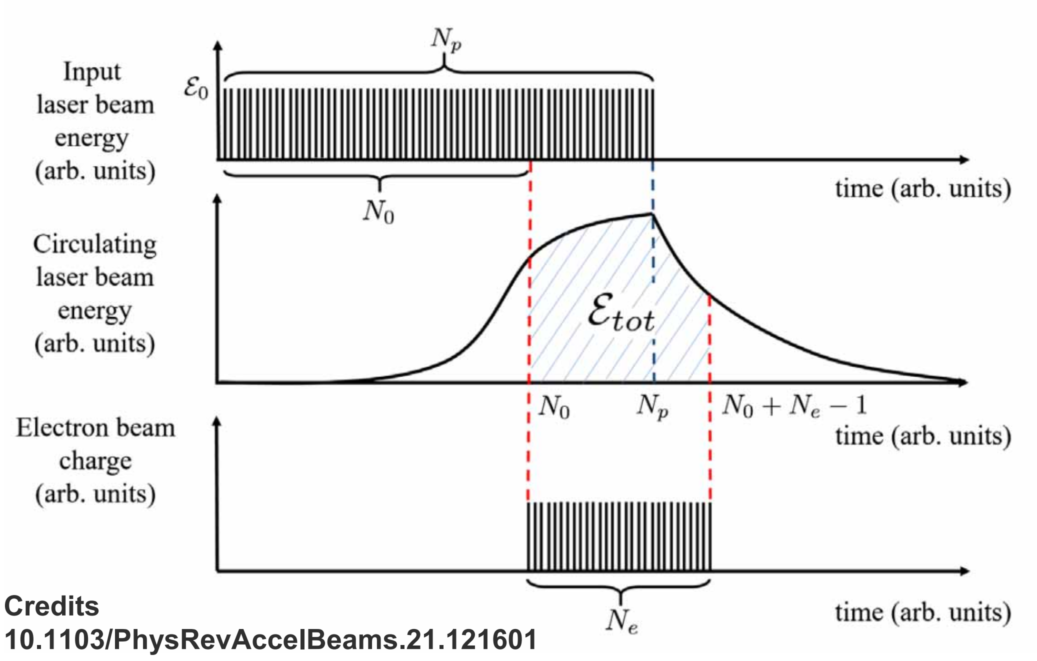


The burst mode operation of a Fabry-Perot cavity:

1. **Has a temporal pattern of the laser pulses similar to the incoming electron train.**
2. **The effective gain is 2 to 3 orders of magnitude larger than for the continuous wave mode.**
3. **Due to the lower intracavity average power, thermal effects on the cavity mirrors are minimised.**

- **A planar bowtie cavity was chosen, providing good stability of the resonator modes.**
- **The electron train interacts with the intracavity laser beam with a crossing angle ϕ.**

Why burst mode operation?



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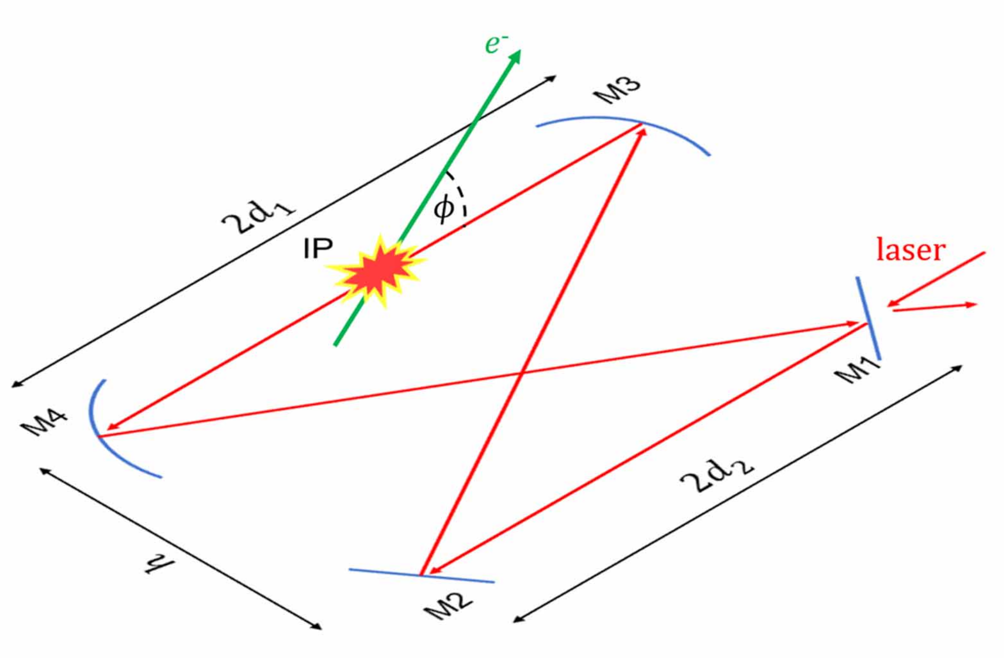
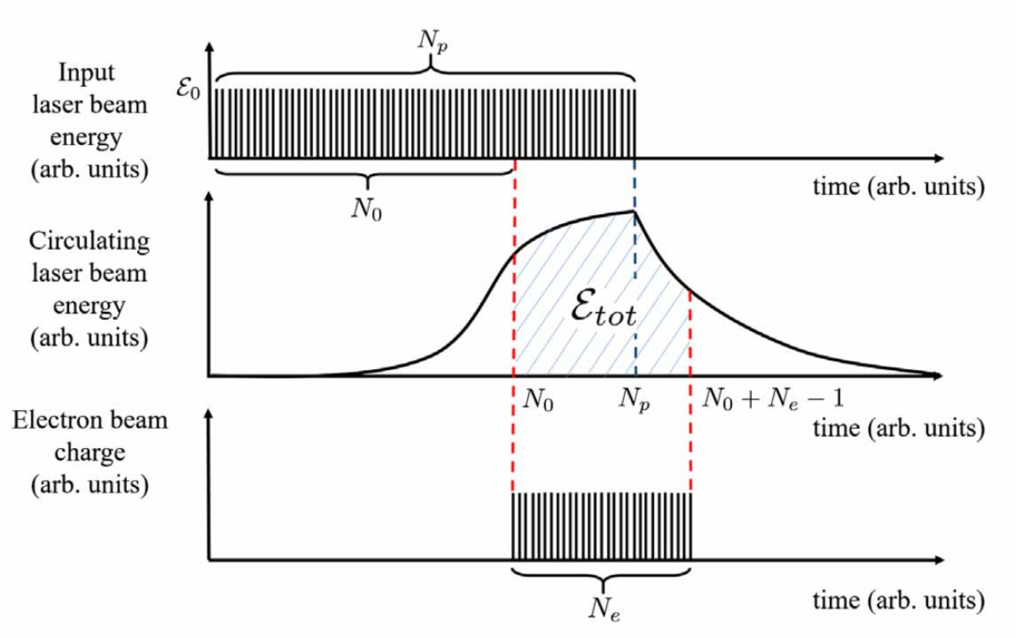


Maximisation of the number of laser photons for the interaction

Optimisation of the burst mode parameters

Minimisation of the laser waist size at the IP & maximisation of the macropulse energy

Optimisation of the geometrical parameters



Burst mode optimisation

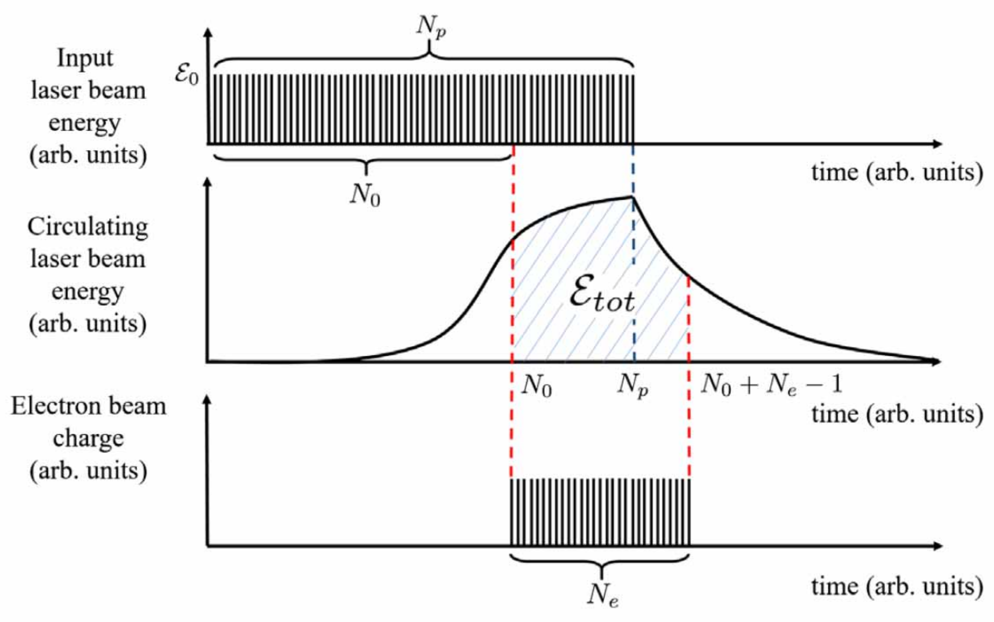
Burst mode FPC optimisation

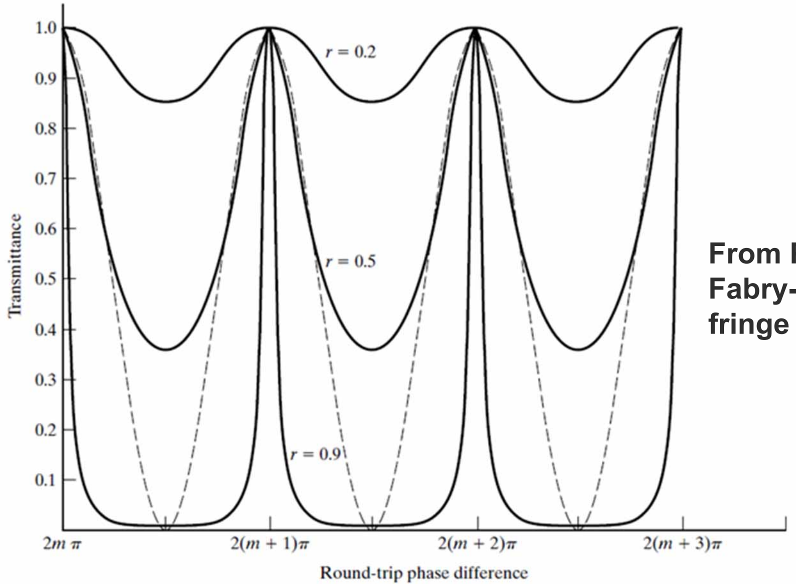
The laser circulating power determines the effective energy

$$\mathcal{E}_{tot} = \epsilon\left(N_p, N_0, N_e, F\right) N_p \mathcal{E}_0$$

- Flux increases linearly with $\mathcal{E}_{tot}$.
- $N_p \mathcal{E}_0 = U$ is limited by the FPC geometry (mirror LIDT).
- Input: the number of electron bunches per train

- A large finesse (*F*) allows for more laser pulses to be stored in the cavity → larger $\mathcal{E}_{tot}$.
- High finesse cavities (e.g., $F > 10000$), require precise laser matching and mirror tuning → restrict the maximum finesse to 1000.





From Pedrotti: Fabry-Perot fringe profile

Optimisation strategy: burst mode parameters

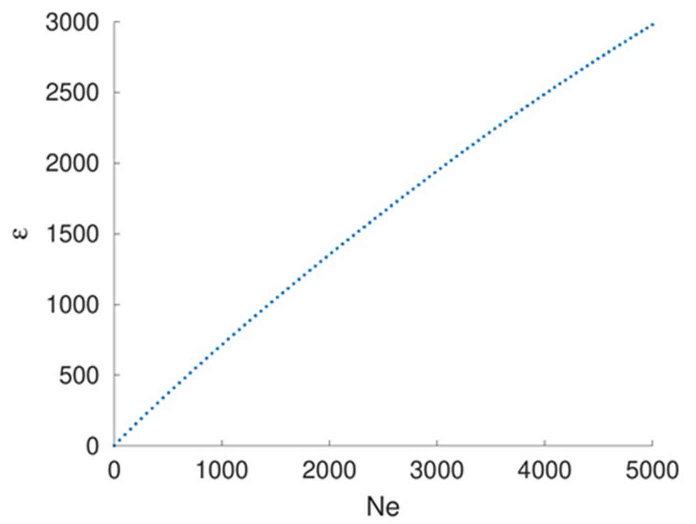
Previous study

$$\mathcal{E}_{tot} = \epsilon(N_p, N_0, N_e, F) N_p \mathcal{E}_0$$

Input the number of electron bunches → Numerically maximise the effective gain → Obtain the burst mode parameters → Derive the input laser parameters

Nb electron bunches

$$\epsilon = \frac{T_1}{(1-r)^2}\left(1 - \frac{N_0}{N_p} + \frac{2r(r_p^N - r_0^N)}{N_p(1-r^2)} + \frac{(1-r_0^N)^2 - r^{2N_e} r^{2N_0}(1-r^{-N_p})^2}{N_p(1-r^2)}\right)$$



Laser input parameters

Previous study: Aurélien Martens, 10.1103/PhysRevAccelBeams.21.121601

Optimisation strategy: burst mode parameters
Current study

$$\mathcal{E}_{tot} = \epsilon(N_p, N_0, N_e, F) N_p \mathcal{E}_0$$

The previous study presents some issues:

1. **The ϵ was maximised by tuning N_p, N_0, and F, although there is a missing N_p term.**
2. **The $\mathcal{E}_0$ is treated as a free variable, although it is fixed by the input laser.**
3. **The maximisation of ϵ does not correspond to a global maximum of $\mathcal{E}_{tot}$.**

Set the nb of electron bunches AND
The laser micropulse energy AND
the maximum finesse

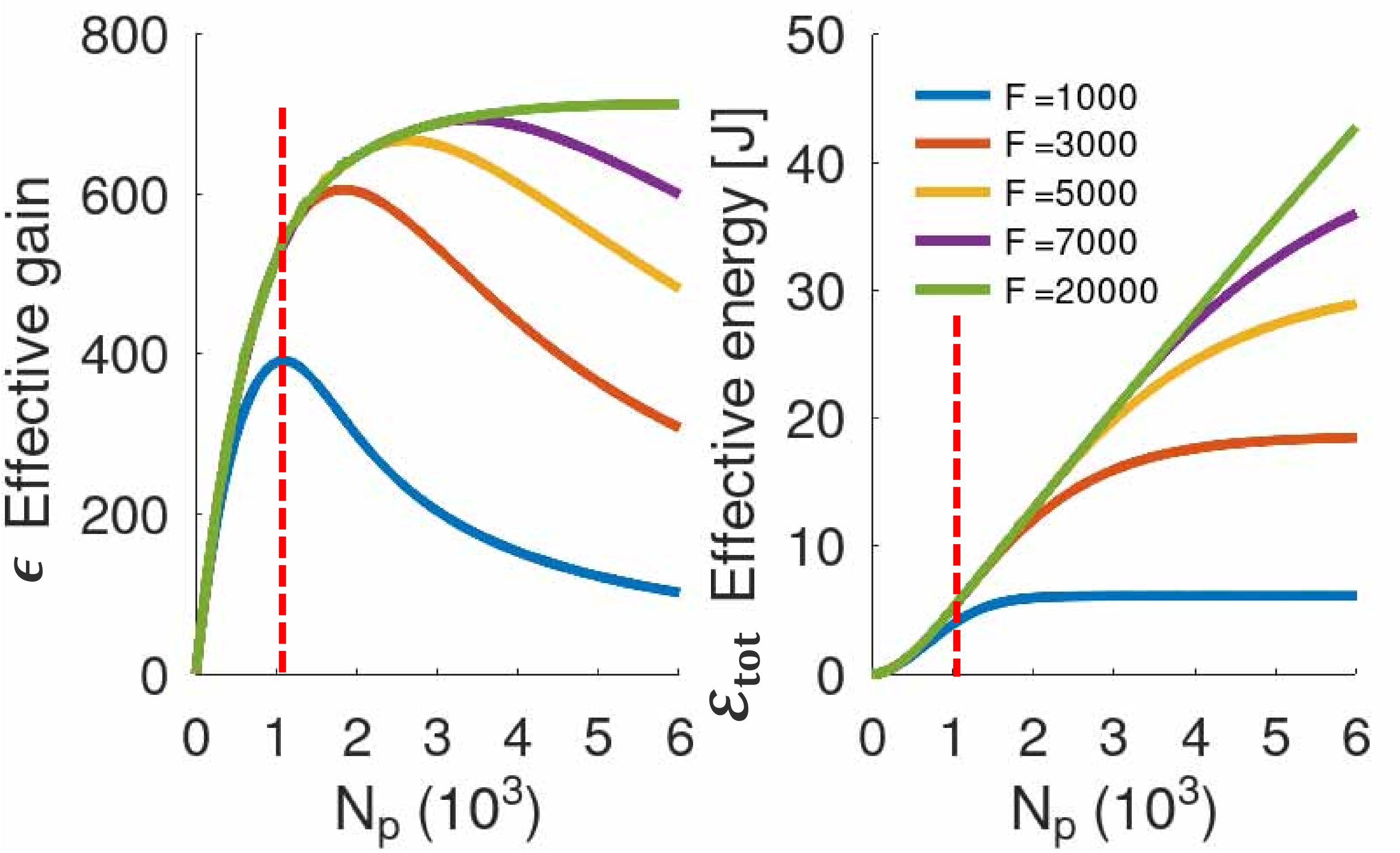
Scan the optimised effective gain for a range of N_p

Obtain a curve for $\mathcal{E}_{tot}$ and identify the maximum value

Obtain burst mode parameters and requirements on the input laser

Parametric scans of the effective energy and the effective gain

$$\mathcal{E}_{tot} = \epsilon(N_p, N_0, N_e, F) N_p \mathcal{E}_0$$

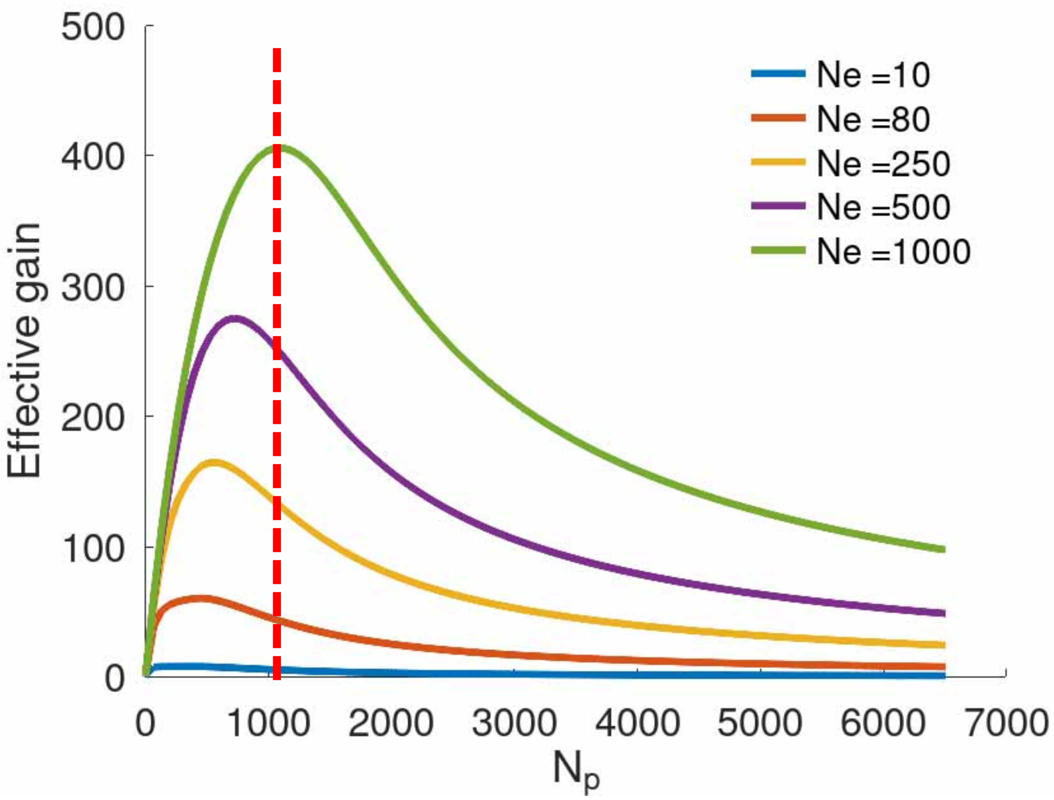


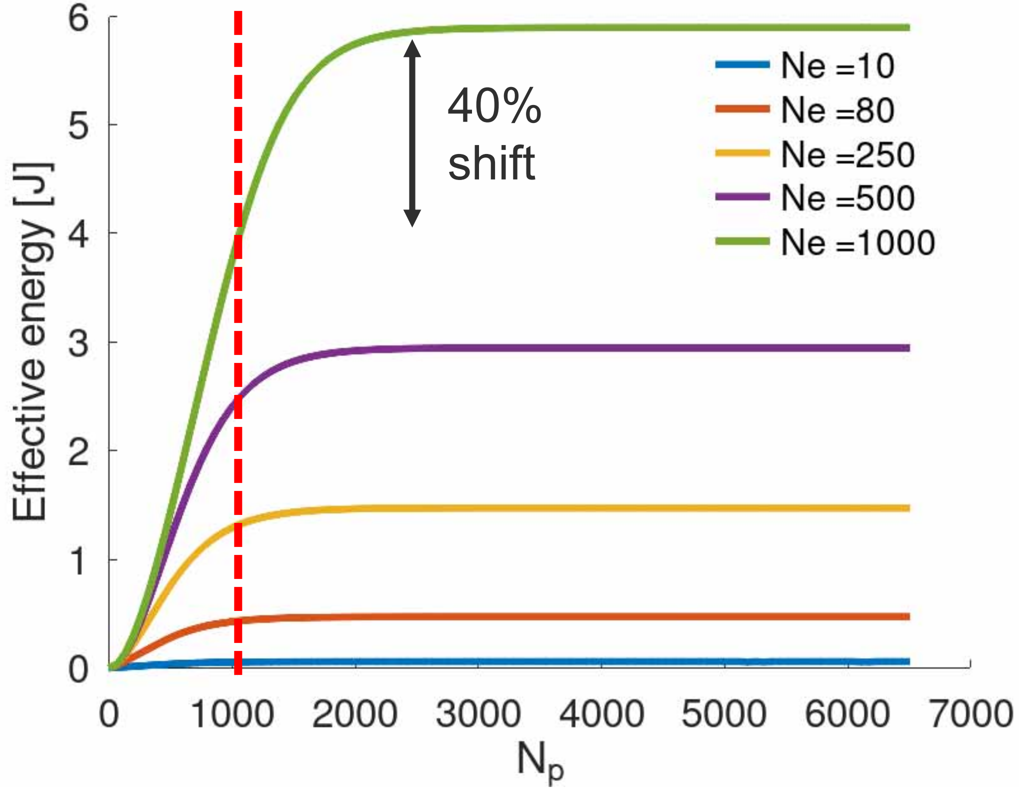
- **For any *F*, ϵ has a local maximum.**
- **However, $\mathcal{E}_{tot}$ continues to increases past the point of maximum ϵ.**
- **The optimisation of the burst mode parameters should maximise $\mathcal{E}_{tot}$ directly.**

Input: $\mathcal{E}_0 = 10\ \mu J$; $N_e = 1000$

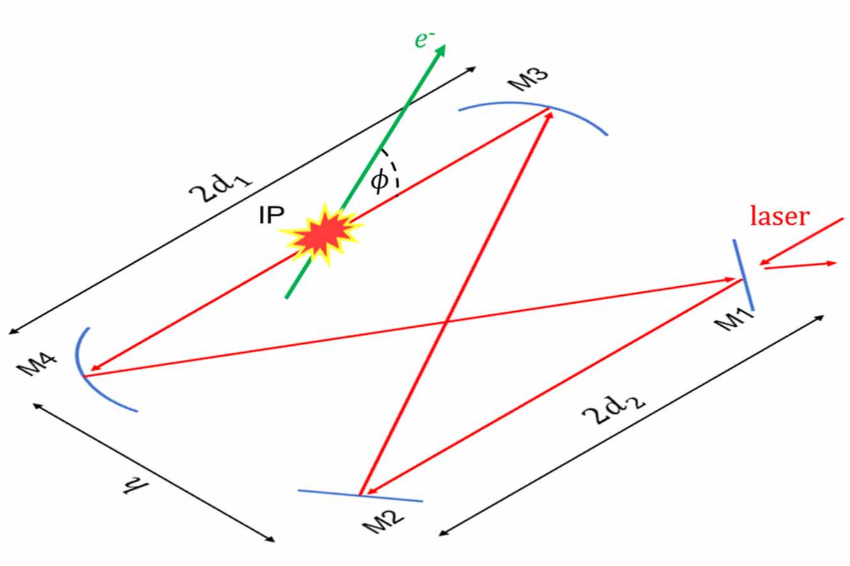
Parametric scans of the effective energy and the effective gain

By choosing the max $\mathcal{E}_{tot}$ instead of the max ϵ, a 40% increase in the effective energy is obtained.





Input: $\mathcal{E}_0 = 10\ \mu J$; $F_{max} = 1000$



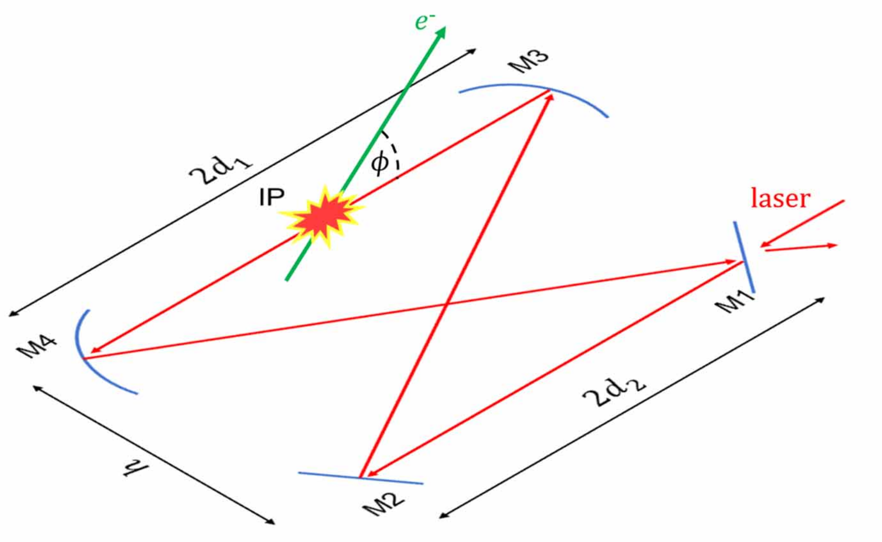
Geometry optimisation

Inputs and requirements

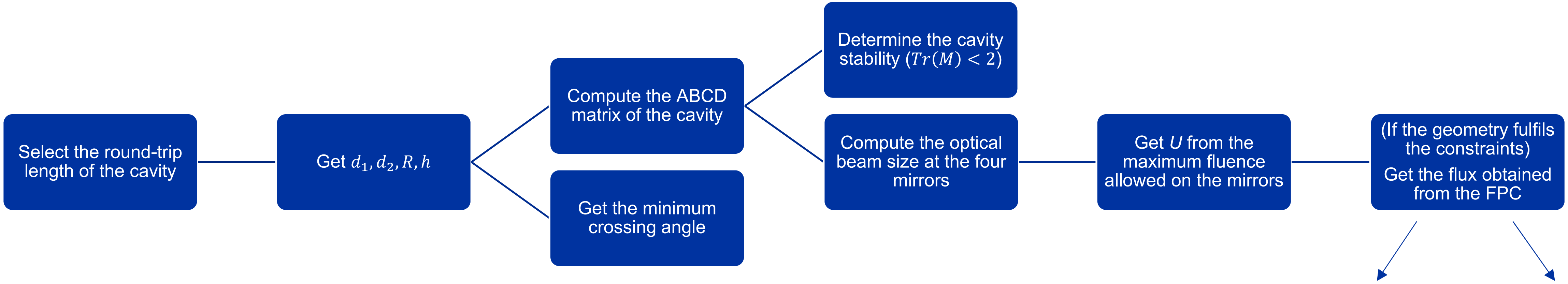
- To maximise flux, Fabry-Perot cavities require a small waist size at the IP, and a large macropulse energy.
- The only input is the cavity round-trip length (set to a harmonic of the laser pulse repetition rate / electron bunch spacing).
- The previous study used a Monte-Carlo technique for the optimisation; we used the simplex algorithm → four orders of magnitude faster convergence.

Parameter	Symbol	Unit
Roundtrip length	L_{RT}	cm
Cavity height	h	cm
Distance between spherical mirrors	$2d_1$	cm
Distance between planar mirrors	$2d_2$	cm
Radius of curvature	R	cm
Mirror diameter	Φ	deg
Crossing angle	ϕ	deg

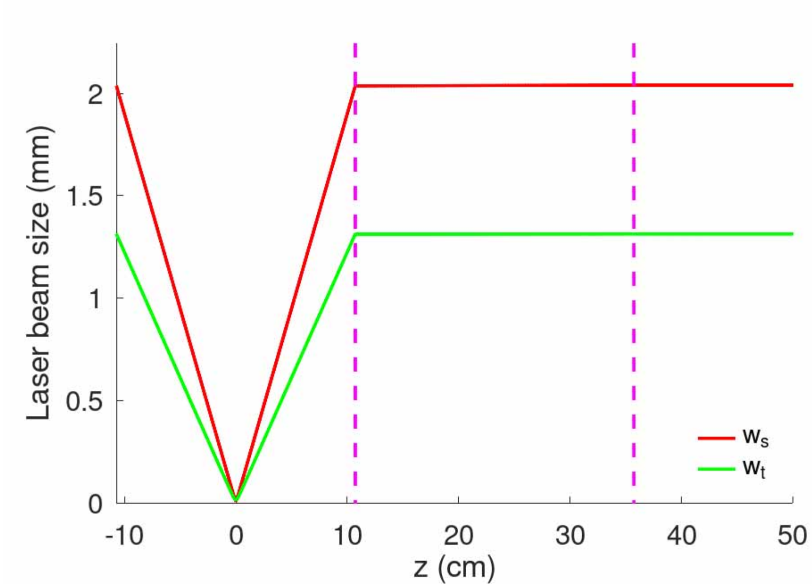
Free parameter	Symbol	Constrain
Spherical mirrors spacing	d_1	$[0, L_{RT}/4]$
Planar mirrors spacing	d_2	
Cavity height	h	
Radius of curvature	R	
Mirror diameter	Φ	$h - \Phi > h\lvert d_1 - d_2\rvert/(d_1 + d_2)$ $d_1 \tan\alpha > \Phi/2$
Roundtrip length	L_{RT}	$n \times c/(f_{\text{rep}}), n \in \mathbb{Z}$
Laser pulse energy	U	$U < \mathcal{F}_{\max}\pi w_s w_t/2$
Cavity stability		$Tr(M_t) < 2$ $Tr(M_s) < 2$

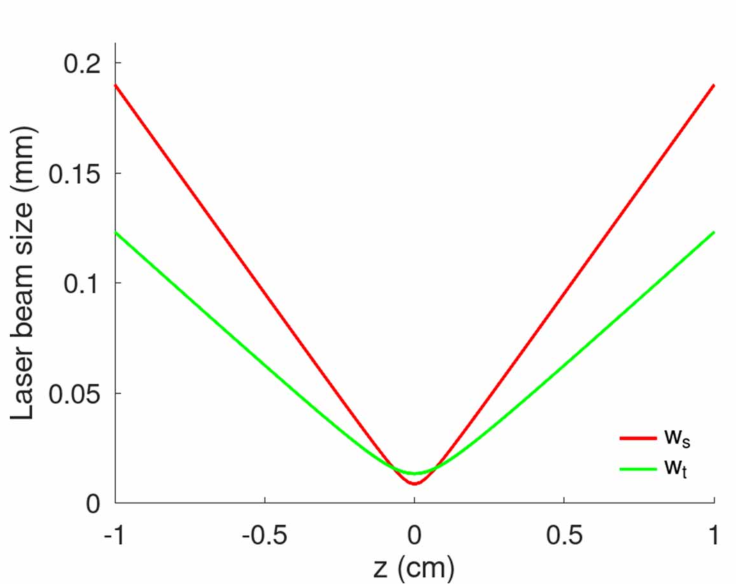


Optimisation strategy: geometrical parameters

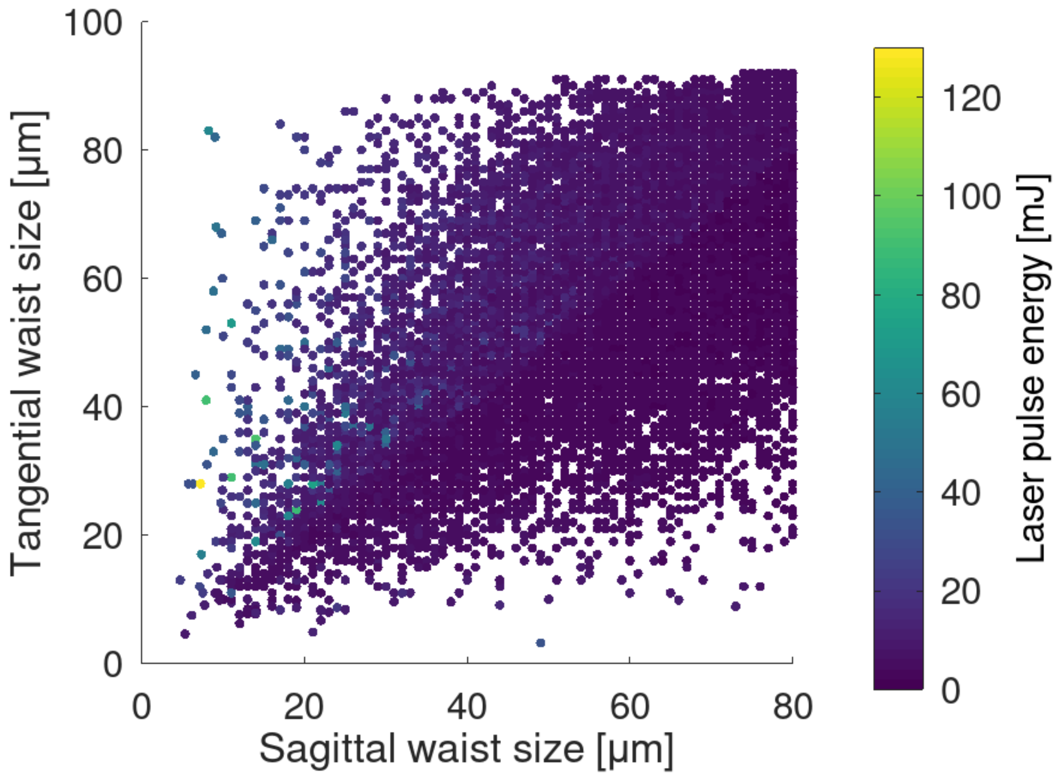


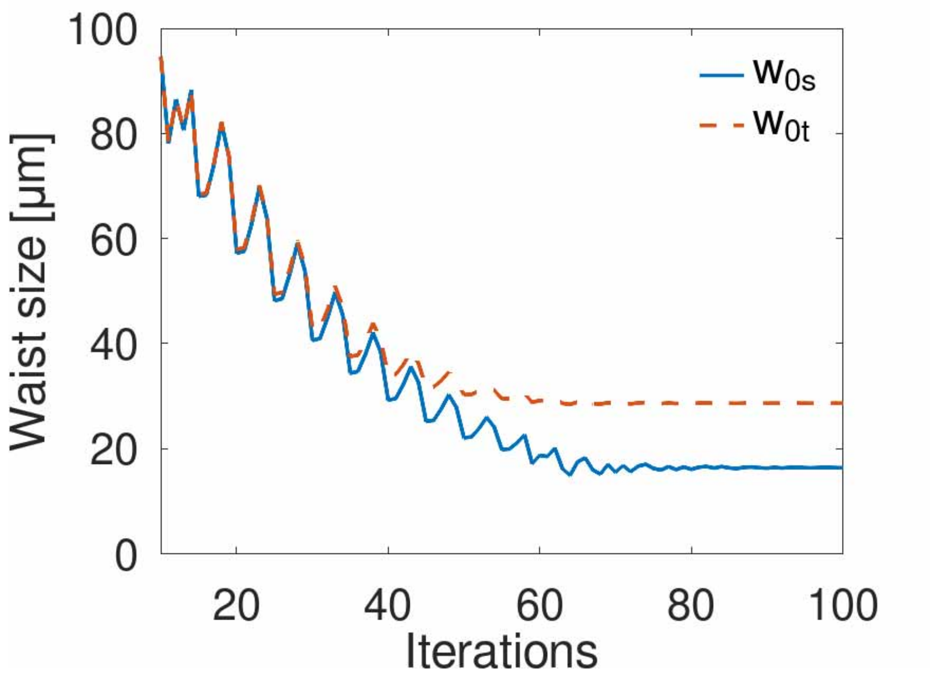
Monte-Carlo (previous study)	Simplex (this study)
Generate millions of randomised geometries	Set a merit function maximising flux under the set constraints
Select configuration with the highest flux	Run the optimisation until it converges





Runtime test of Monte-Carlo vs Simplex





Monte-Carlo (previous study)	Simplex (this study)
> 10 million iterations for convergence	< 100 iterations for convergence

The simplex optimisation is more than four orders of magnitude faster than the Monte-Carlo technique

Impact of thermal effects

Thermal lensing

- **The performance of the Fabry-Perot cavity can be affected by thermal effects, such as thermal lensing.**

- **Main contribution to thermal lensing is from the temperature dependence of the refractive index of the optical substrate**

$$f_{\text{thermal}} = \frac{2\pi\kappa}{1.3b(dn/dT)l}\frac{w^2}{P} = \frac{1}{m_0}\frac{w^2}{P}$$

Note: m_0 is a material-dependent parameter.

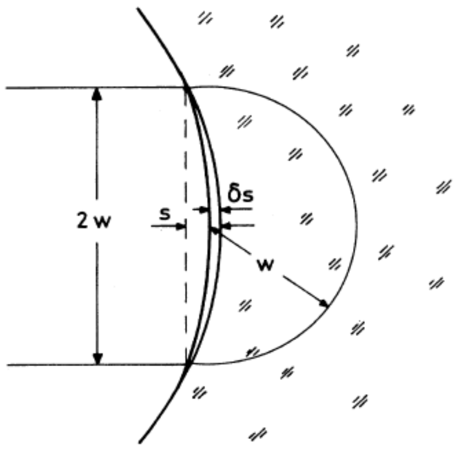


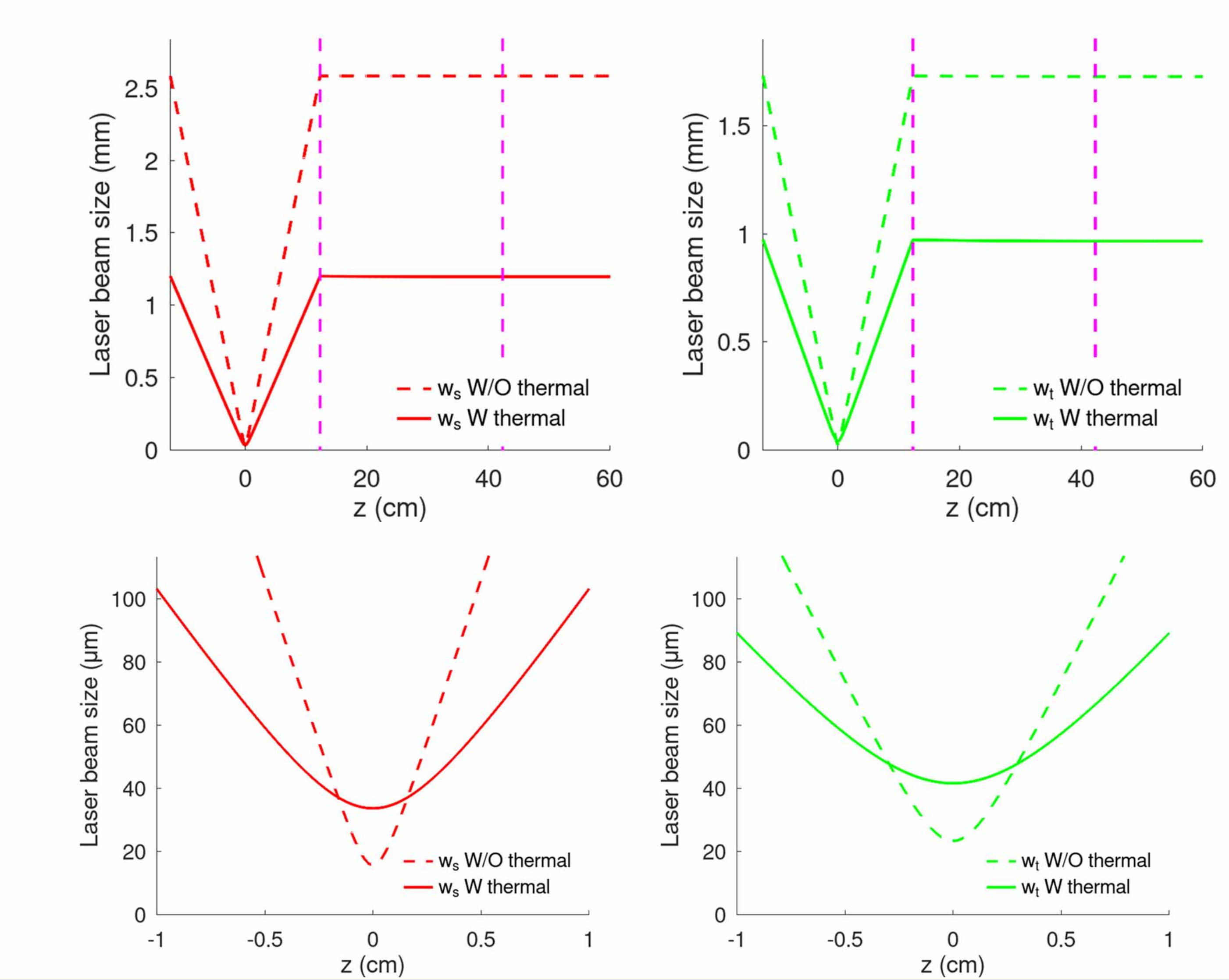
FIG. 1. Absorption of light at the coated surface of optical components is responsible for a temperature gradient across a hemisphere around the reflection spot with radius w. The related thermal expansion of the substrate changes the sagitta s by δs.

From 10.1103/PhysRevA.44.7022

ABCD matrix w/o thermal effects → Get the laser beam size on the 4 mirrors → Get f_{thermal} for each mirror → ABCD matrix w thermal effects → Get waist size at the IP and *U*

Parameter	Symbol	Unit
Beam power	*P*	W
Beam waist	*w*	M
Thermal conductivity	κ	W/(m K)
Temperature dependence of the refractive index	*dn/dT*	1/K
Thickness of the medium	*l*	m
Absorption coefficient	*b*	1/m

Implementation of thermal effects



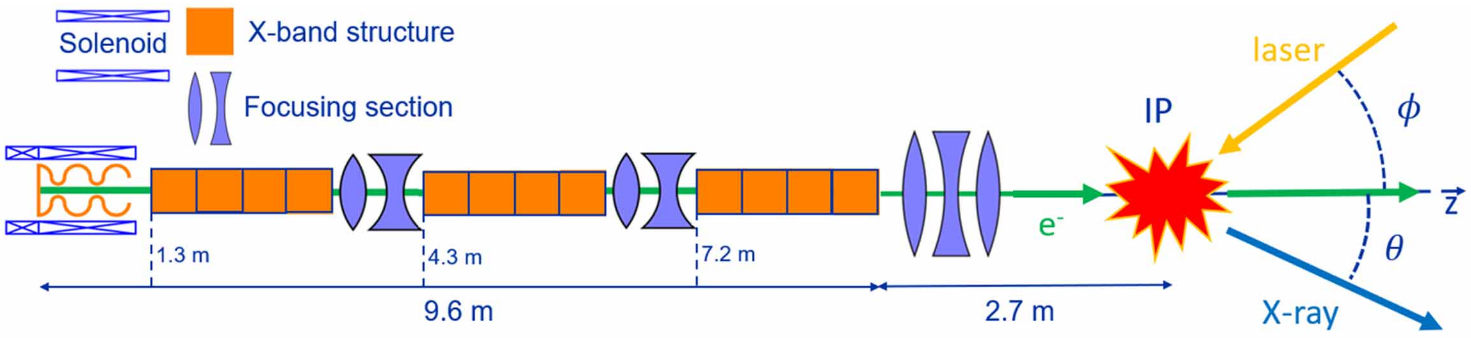
- **Thermal effects were implemented for a cavity with FS substrates.**
- **The thermal focal length is typically in the 100 - 1000 m range.**
- **Flux decrease up to 83% from adding thermal effects to a pre-optimised cavity**
- **Effect can be cancelled by running the optimisation with the thermal effects.**

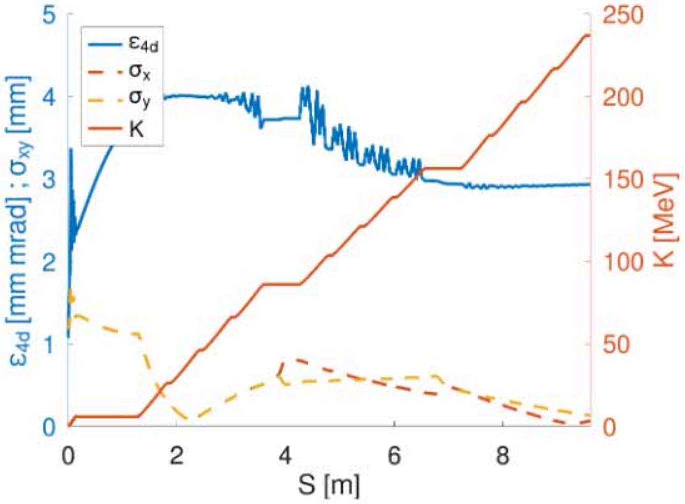
Parameter	Symbol	W/O thermal lensing	W thermal lensing
Waist size sagittal	w_{0s} [μm]	15.6	33.6
Waist size tangential	w_{0t} [μm]	23.3	41.5
Pulse energy	U [mJ]	140	36

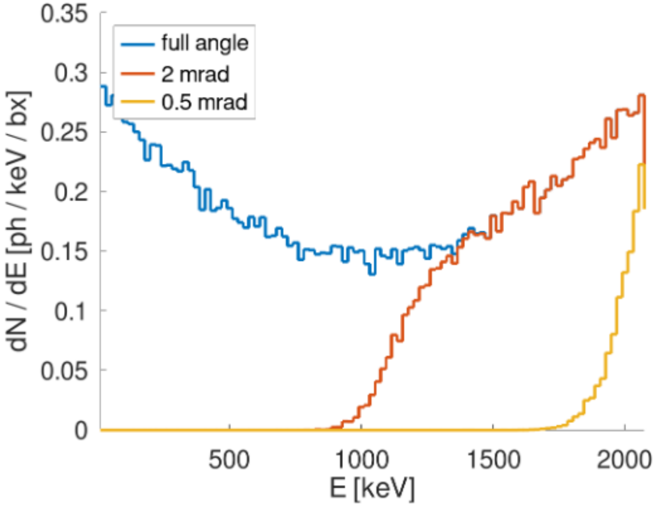
83% loss in flux

Example case: HPCI

Baseline Parameters of a High-Pulse Current Injector (HPCI) ICS source (optimised Fabry-Perot cavity)







Parameter	Value	Unit
Round-trip length	1	m
$2d_1$, $2d_2$, h, R	21.43, 28.54, 1.16, 21.46	cm
N_p, N_0, F	2292, 1298, 1000	
w_{0s}/w_{0t}	8.6/13.4	μm
Effective gain	264	
Effective energy	6	J

Colliding Laser	Value	Unit
Wavelength	515	nm
Pulse energy	10	μJ
Pulse length	1.2	ps
Crossing angle	2	°

Outcoming Photons	Value	Unit
Compton edge	2.1	MeV
Total flux, $\mathcal{F}$	2.2×10^{13}	ph/s
Bandwidth (0.5 mrad)	2.0	%
Flux (0.5 mrad)	1.6×10^{12}	ph/s
Average Brilliance, $\mathcal{B}$	4.4×10^{13}	(1)
Peak Brilliance, $\hat{\mathcal{B}}$	3.9×10^{23}	(1)

(1) ph/(s mm^2 mrad^2 0.1%BW)

See talk on Thursday 2 pm by A. Latina, TH4A2

Conclusions

Significant improvements were made for the optimisation of a burst-mode operated Fabry-Perot cavity.

The goal of the optimisation is to provide the maximum possible flux, with state-of-the-art technology.

***Burst parameters optimisation*: By maximising the effective energy of the cavity, an increase in the total flux of more than 40% was obtained.**

***Geometry optimisation*: By using the simplex algorithm, the computation runtime was reduced by more than four orders of magnitude. Thermal effects were implemented in the geometrical optimisation.**

***Example case*: The present considerations can be used to design high-intensity linac-based ICS sources. By applying this optimization to the HPCI source, a total flux of 10^{13} ph/s could be obtained.**