DFCSR: A Fast Calculation of 2D/3D Coherent Synchrotron Radiation in Relativistic Beams

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U.S. DEPARTMENT OF ENERGY | Stanford University

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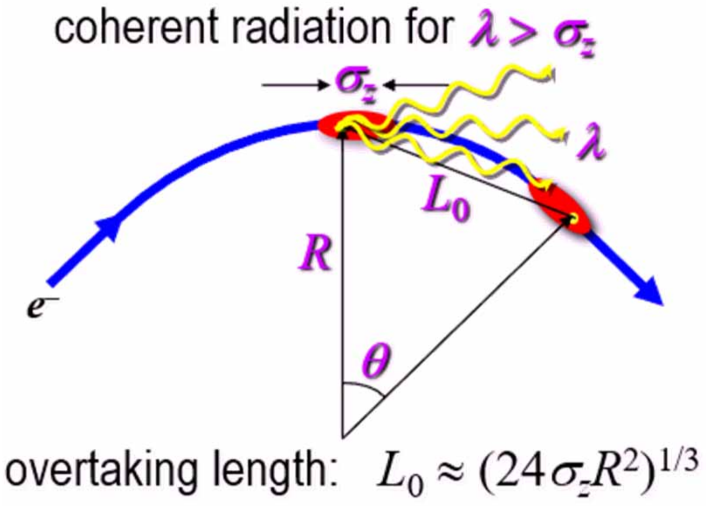
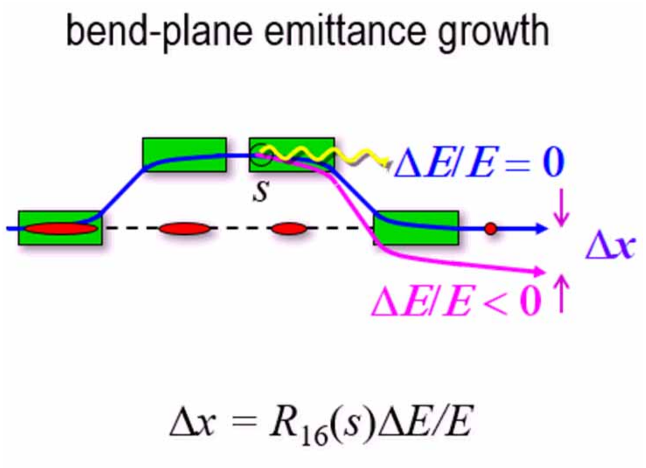
Introduction

- When the trajectory of a relativistic beam is bent by magnetic field, the beam radiates electromagnetic field. It can radiate coherently for very short bunch if the wavelength of the synchrotron radiation exceeds the length of the bunch (CSR).
- CSR can increase energy spread and transverse emittance. It is one of the most important limits to the brightness of the electron beam in storage rings, free electron lasers (FEL) and high-energy linear colliders

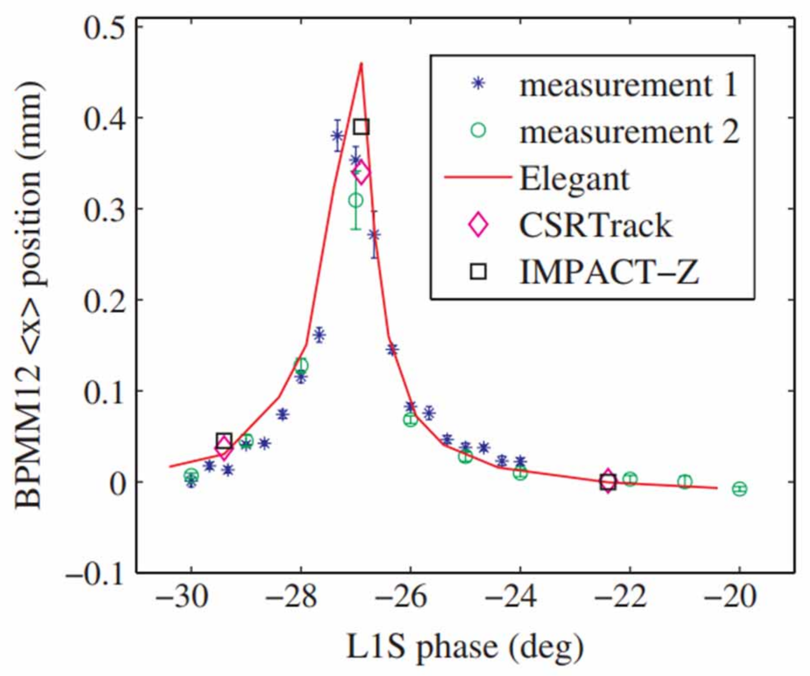


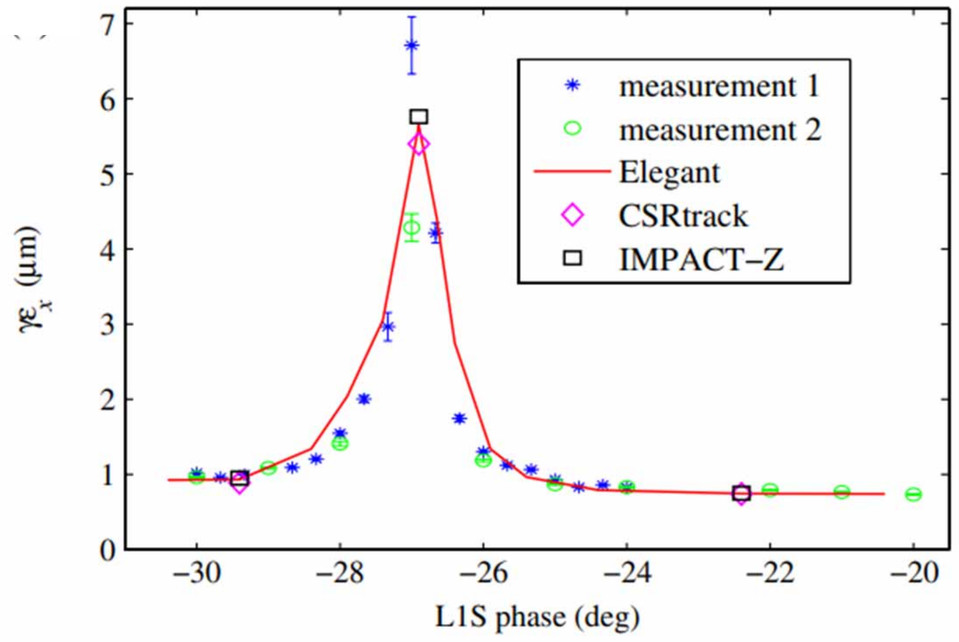


Paul Emma. "Acceleration and Compression of High Brightness Electron Beams in Linac-Based FELs". 2015

Introduction

There are well-established 1D models of the CSR wake. 1D models neglects transverse dimension of the beam, and they agree well with experimental results when the beam current is relatively low.

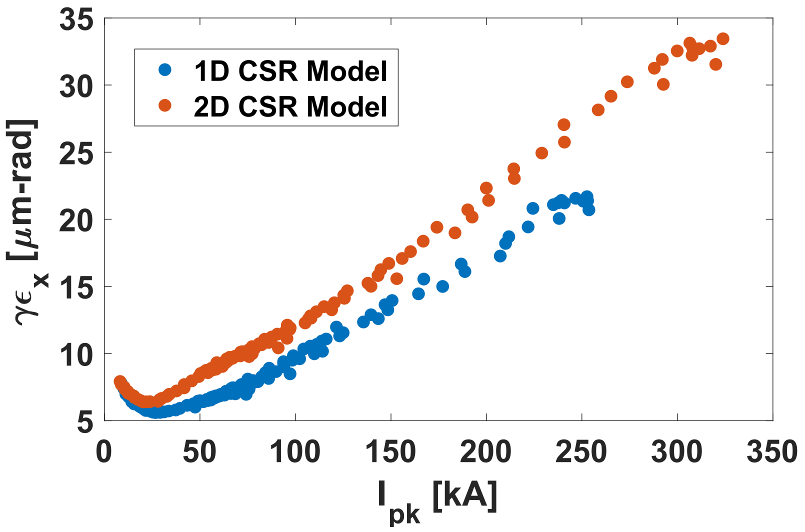




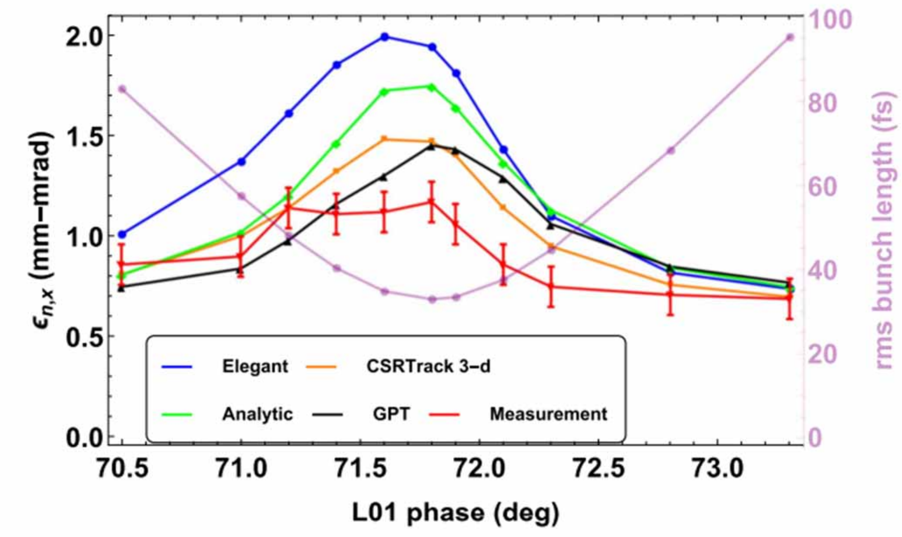
K.L.F. Bane et al. Phys. Rev. ST Accel. Beams, 12, 030704 (2009).

Introduction

- 1D models become invalid when Derbenev criterion is not satisfied, i.e. beam with large transverse extension and small bunch length. $\sigma_x(\rho\sigma_z^2)^{-1/3} \ll 1$
- The departure from 1D models have been explored both in simulation and experiment



G. White, FACET-II Science Workshop October 2017;



Brynes, A. D., et al. New Journal of Physics 20.7 (2018): 073035.

- Generation of ultra-short, high-brightness electron bunches may lead to a parameter space that 1D CSR models fail to work.
- A fast and reliable 2D/3D CSR model is required for the next generation of accelerators and light sources

Introduction

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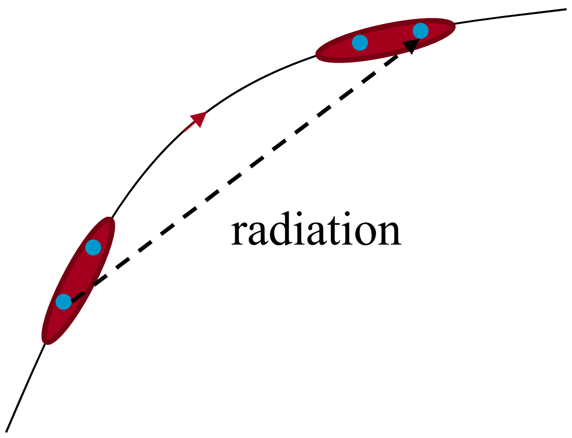
- Difficulties in 2D/3D CSR computation
  - history-dependent nature of CSR
  - difficult 3D retardation conditions
  - scaling of self-interactions in computation.

- 2D/3D CSR codes being actively studied[1]
  - *CSRtrack/TraFiC4 (2005)*
  - *LW (2013)*
  - *GPT-CSR (2018)*
  - *PyCSR (2020)*
  - *Lucretia 2D CSR (2020)*
  - *CoSyR(2021)*



DFCSR

- a CSR code based on the history of density functions of electron beams.
- Accurate and efficient computation of 2D/3D CSR
- Open-sourced and user-friendly

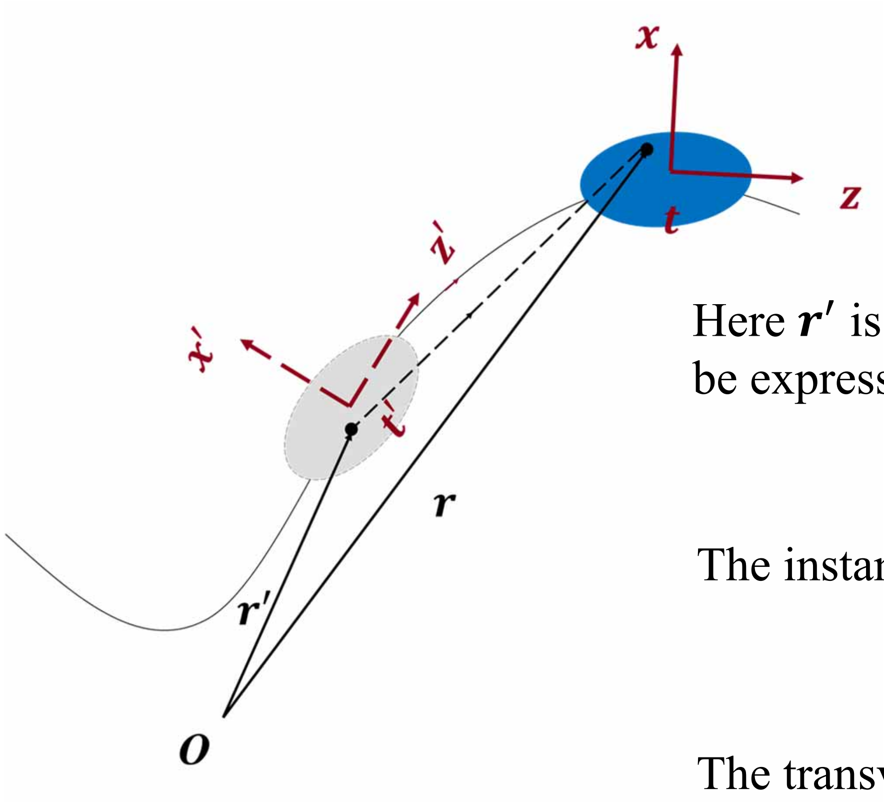
1. Mayes, C. E. *Journal of Instrumentation* 16.10 (2021): P10010.

Outline

- CSR Model and Algorithm
- Benchmarks
  - Dipole
  - Berlin Benchmark Chicane
  - FACET-II Benchmark Chicane
- Computational Efficiency
- An Example of Using Our Open-sourced Codes

• CSR Model and Algorithm

Electron beam is described by its time-dependent density functions of charge $\rho(\boldsymbol{r},t)$ and velocity $\boldsymbol{v}(\boldsymbol{r},t)$. Retard potentials are given by



$$\phi(\boldsymbol{r},t)=\int d^3r'\frac{\rho(\boldsymbol{r}',t_{\text{ret}}(\boldsymbol{r},\boldsymbol{r}',t))}{|\boldsymbol{r}'-\boldsymbol{r}|}$$

$$\boldsymbol{A}(\boldsymbol{r},t),=\frac{1}{c}\int d^3r'\frac{\boldsymbol{v}(\boldsymbol{r}',t_{\text{ret}}(\boldsymbol{r},\boldsymbol{r}',t))\rho(\boldsymbol{r}',t_{\text{ret}}(\boldsymbol{r},\boldsymbol{r}',t))}{|\boldsymbol{r}'-\boldsymbol{r}|}$$

Here $\boldsymbol{r}'$ is retarded space coordinates. $t_{ret}=t-|\boldsymbol{r}-\boldsymbol{r}'|/c$. The electric field can be expressed as

$$\boldsymbol{E}(\boldsymbol{r},t)=-\boldsymbol{\nabla}_{\boldsymbol{r}}\phi(\boldsymbol{r},t)-\frac{1}{c}\frac{\partial\boldsymbol{A}(\boldsymbol{r},t)}{\partial t}$$

The instantaneous energy change per unit time and per unit charge (longitudinal force) is

$$\mathcal{P}(\boldsymbol{r},t)=\boldsymbol{v}(\boldsymbol{r},t)\cdot\boldsymbol{E}(\boldsymbol{r},t)$$

The transverse force is given by

$$\boldsymbol{F}_\perp=\boldsymbol{E}_\perp+\boldsymbol{\beta}\times\boldsymbol{B}=-\nabla_\perp\phi-\frac{1}{c}\left.\frac{\partial\boldsymbol{A}}{\partial t}\right|_\perp+\boldsymbol{\beta}\times\nabla\times\boldsymbol{A}\,.$$

[1] Stupakov, G., & Tang, J. (2021). Physical Review Accelerators and Beams, 24(2), 020701.
[2] Stupakov, G. Physical Review Accelerators and Beams 25.1 (2022): 014401.

• CSR Model and Algorithm

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With further derivations described in [1], the longitudinal wakes can be expressed by

$$\mathcal{P}(\boldsymbol{r},t) = -c\int \frac{d^3r'}{|\boldsymbol{r}'-\boldsymbol{r}|}\left[\boldsymbol{\beta}(\boldsymbol{r},t) - (\boldsymbol{\beta}(\boldsymbol{r},t)\cdot\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret}))\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret})\right]\cdot\partial_{\boldsymbol{r}'}\rho(\boldsymbol{r}',t_{\rm ret})$$
$$+ c\int \frac{d^3r'}{|\boldsymbol{r}'-\boldsymbol{r}|}(\boldsymbol{\beta}(\boldsymbol{r},t)\cdot\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret}))\rho(\boldsymbol{r}',t_{\rm ret})\partial_{\boldsymbol{r}'}\cdot\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret})$$
$$- \int \frac{d^3r'}{|\boldsymbol{r}'-\boldsymbol{r}|}\rho(\boldsymbol{r}',t_{\rm ret})\boldsymbol{\beta}(\boldsymbol{r},t)\cdot\partial_{t_{\rm ret}}\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret}),$$

Similarly, we can get the transverse CSR wake [2]

$$F_\perp = \int \frac{ds'dx'\rho}{|\boldsymbol{r}'-\boldsymbol{r}|^3}(\boldsymbol{r}-\boldsymbol{r}')\cdot(\boldsymbol{n}-\boldsymbol{n}')$$
$$+ \frac{1}{c}\int \frac{ds'}{|\boldsymbol{r}'-\boldsymbol{r}|^2}\partial_{t_{\rm ret}}\lambda(\boldsymbol{r}-\boldsymbol{r}')\cdot(\boldsymbol{n}-\boldsymbol{n}') - \frac{1}{c}\int\frac{ds'}{|\boldsymbol{r}'-\boldsymbol{r}|}\boldsymbol{n}\cdot\boldsymbol{\tau}'\partial_{t_{\rm ret}}\lambda.$$

- $\boldsymbol{\beta}(\boldsymbol{r},t), \rho(\boldsymbol{r},t)$ and their derivatives can be easily vectorized and parallelized for fast calculation.
- No derivative with respective to time, allowing larger time step
- integrable singularity at $\boldsymbol{r} = \boldsymbol{r}'$

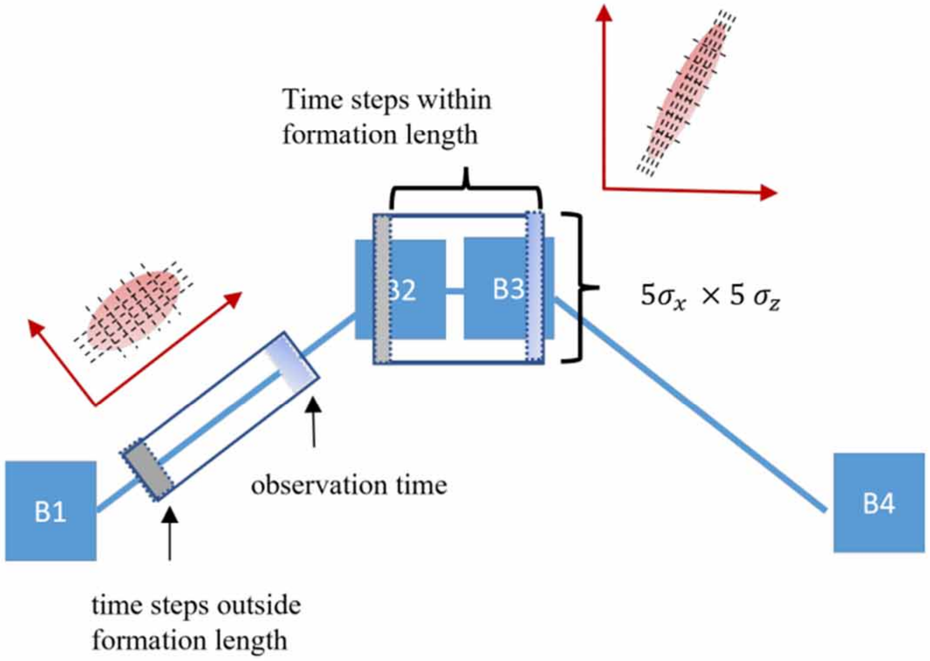
$\boldsymbol{n}$ and $\boldsymbol{\tau}$ are the normal and tangential vector of the reference trajectory.

[1] Stupakov, G., & Tang, J. (2021). Physical Review Accelerators and Beams, 24(2), 020701.
[2] Stupakov, G. Physical Review Accelerators and Beams 25.1 (2022): 014401.

8

CSR Model and Algorithm

- *Deposition, Interpolation, Integration*



Input: lattice, initial beam, step size, grid size;
Initialization: Sample N particles from the beam parameters or read from the beam file.
Compute the reference trajectory $r_0(s)$, $\vec{n}(s)$, $\vec{\tau}(s)$. Set t = 0, history = Empty queue;
while $t \leq L_{lattice}$ **do**

1. Deposit particles in x and z to get $\rho(x, z, t)$ and $v_x(x, z, t)$ in curvilinear frame;
2. Compute derivatives of the density function to get $\frac{\partial\rho(x,z,t)}{\partial x}$, $\frac{\partial\rho(x,z,t)}{\partial z}$, $\frac{\partial v_x(x,z,t)}{\partial x}$ and $\frac{\partial v_x(x,z,t)}{\partial z}$ in curvilinear frame;
3. Append the density functions and their derivatives to the head of history;
4. Truncate the tail of history if longer than formation length;
5. Define integration grids;
6. Do 3D interpolation to get $\rho(x', z', t_{ret})$, $v_x(x', z', t_{ret})$ and their local spatial derivatives on the intergration grids;
7. Define wake grids;
8. Compute longitudinal and transverse CSR wakes on the wake grids;
9. Apply the energy and momentum kicks and advance the particles.

end

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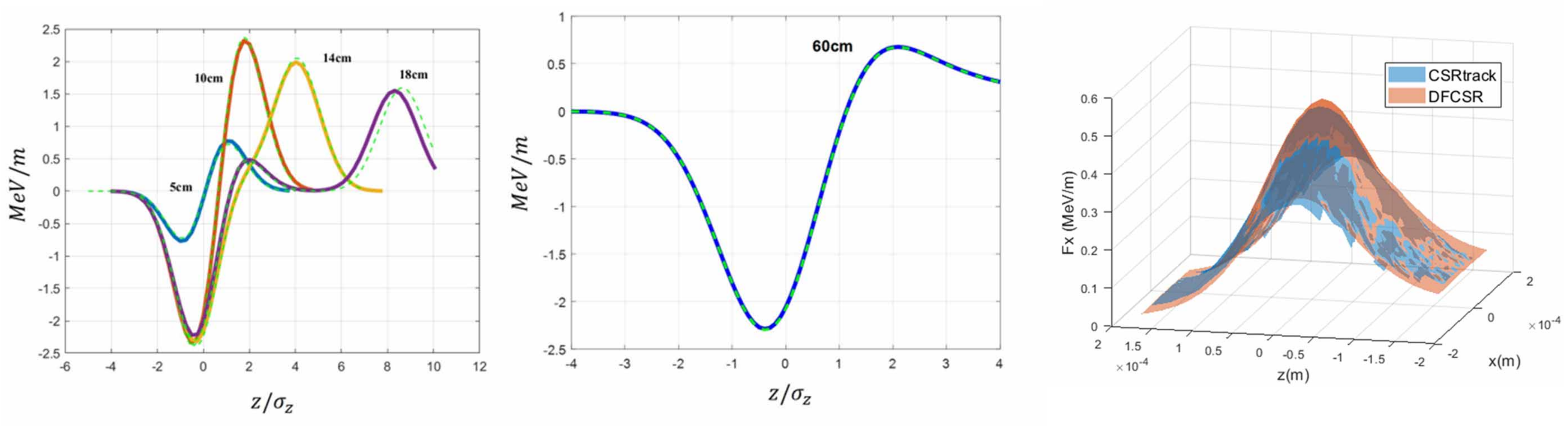
A Gaussian Beam in a Dipole

CSR wake of a Gaussian ebeam in a bending magnet, benchmarked with 1D CSR model and CSRtrack

$$\sigma_x = \sigma_z = 50\ \mu m, R = 1.5m, E = 5GeV$$

$$\sigma_y = 1\mu m$$



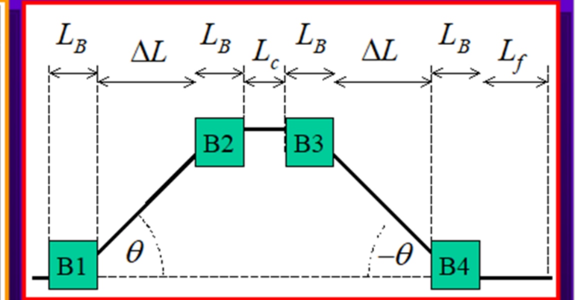
1D model from: Saldin, Schneidmiller, Yurkov, NIMA, A398, 373, (1997); G. Stupakov and P. Emma, EPAC, p. 1479, (2002).

Berlin Benchmark Chicane

We benchmarked our code for a bunch compressor formulated at the CSR workshop at DESY-Zeuthen in 2002.

Chicane CSR Test-Case

Chicane parameters	symbol	value	unit
Bend magnet length (not curved length)	L_B	0.5	m
Drift length (projected; B1-B2 & B3-B4)	ΔL	5	m
Drift length (B2-B3)	L_c	1.0	m
Post-chicane drift length (after B4)	L_f	2.0	m
Bend angle per dipole magnet	$\|\theta\|$	2.770	deg
Bend radius of each dipole magnet	$\|R\|$	10.35	m
Momentum compaction factor	R_{56}	–25.0	mm
2^{nd}-order momentum compaction factor	T_{566}	+37.5	mm
Total projected-length of chicane	L_{tot}	13.0	m
Vertical half-gap of bend magnets	g	12.5	mm



Electron beam parameters	symbol	value	unit
Nominal energy	E_0	5.0	GeV
bunch charge	q	0.5 & 1.0	nC
Incoherent rms relative energy spread	$(\Delta E/E_0)_{u\text{-rms}}$	2.0	10^{-6}
Linear energy-z correlation	a	+36.0	m^{-1}
Total initial rms relative energy spread	$(\Delta E/E_0)_{rms}$	0.720	%
Initial rms bunch length	σ_{zi}	200	μm
Final rms bunch length	σ_{zf}	20	μm
Initial rms norm. emittances	$\gamma\varepsilon_{x,y}$	1.0, 1.0	μm
Initial beta-functions at 1^{st} bend entrance	$\beta_{x0,y0}$	40, 13	m
Initial α-functions at 1^{st} bend entrance	$\alpha_{x0,y0}$	+2.6,	

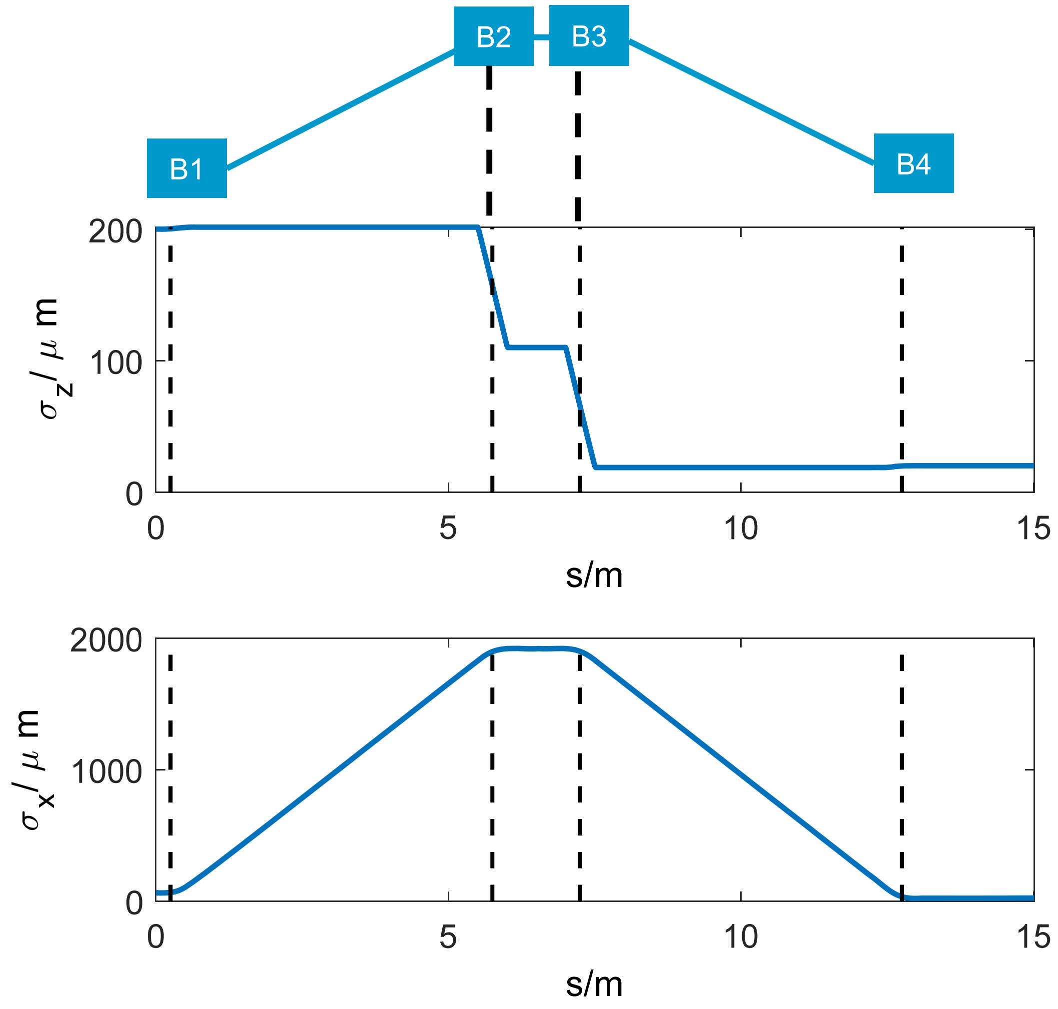
Use line-charge CSR $\gamma \rightarrow \infty$ transient model described in **LCLS-TN-01-12…** (Stupakov/Emma, Dec. 2001) [same now used in ***Elegant***]

…based on **TESLA-FEL-96-14** (Saldin et al., Nov. 1996)

(T_{566} included, no ISR* added)

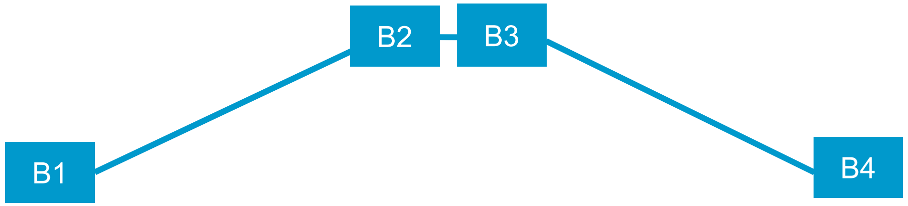
* incoherent synchrotron radiation

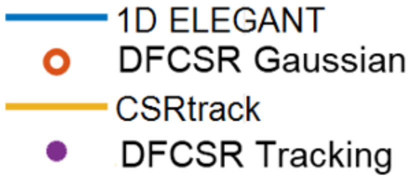
Berlin Benchmark Chicane



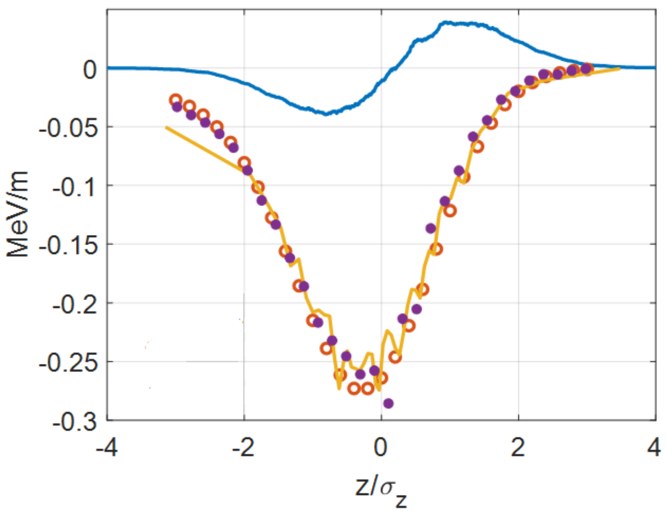
- In the middle of the chicane (B2, B3), the transverse size of the bunch becomes much larger than the longitudinal one. We expect largest deviation from 1D model there.

Berlin Benchmark Chicane

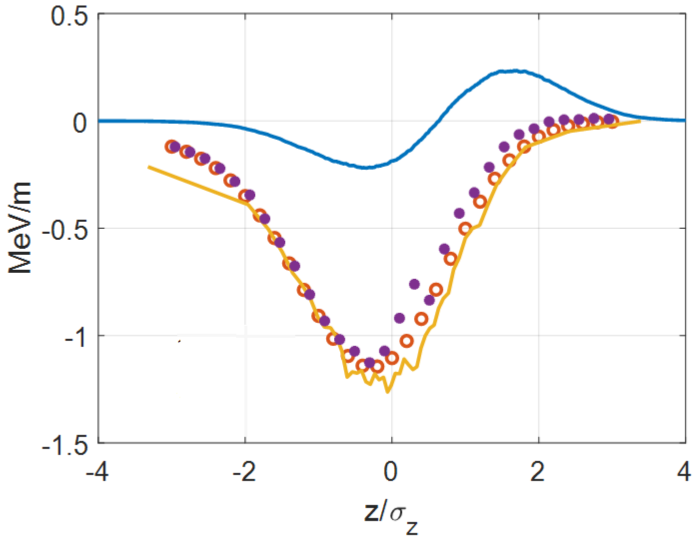




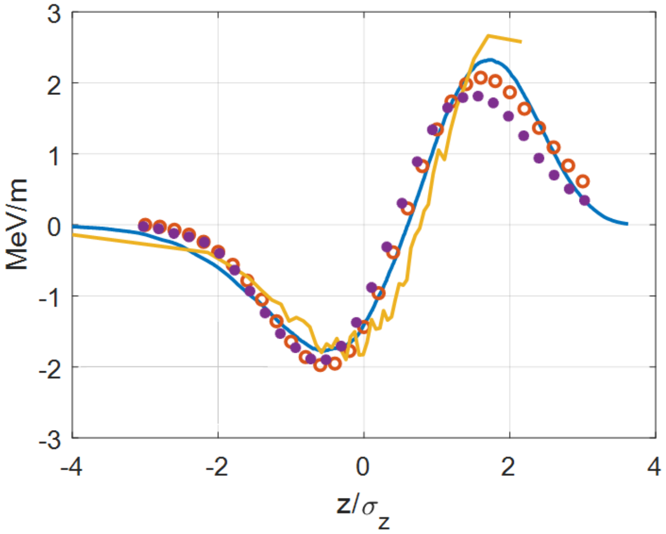
B2



B3

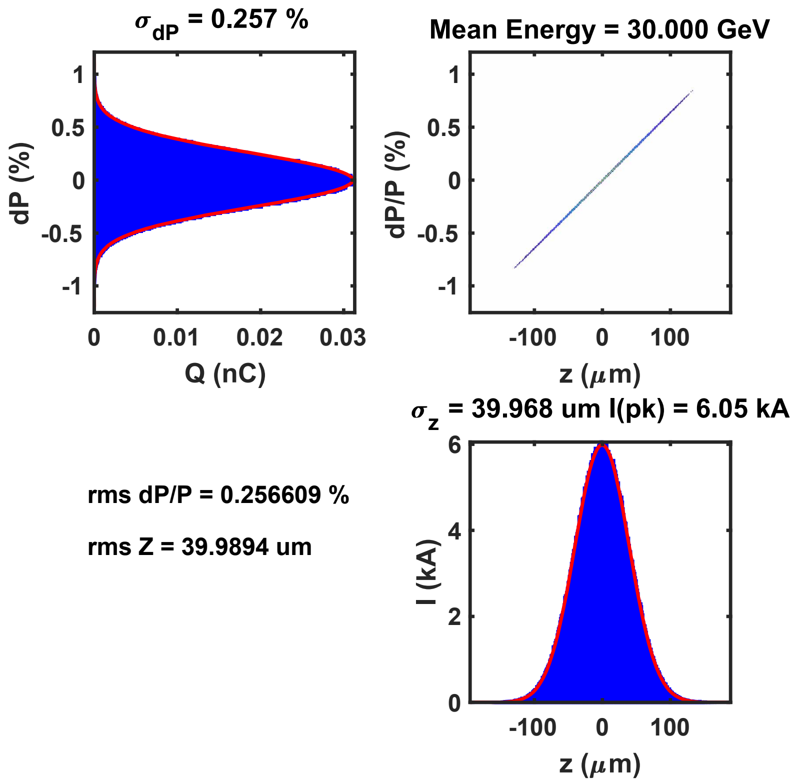


B4



FACET-II Benchmark Chicane

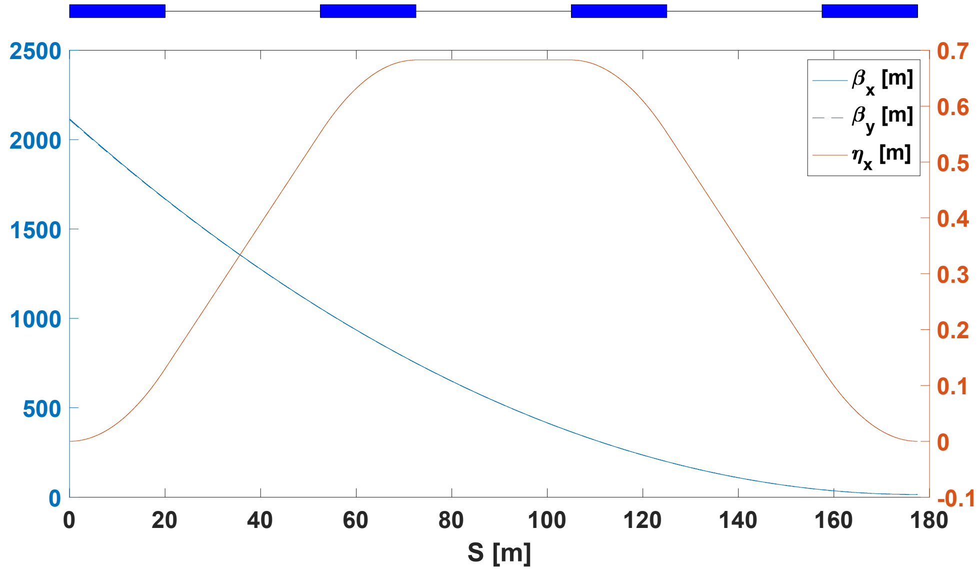
Initial Longitudinal Profile



dE/dz chirp = 1925 (GeV/m)

σ_z = 40 μm

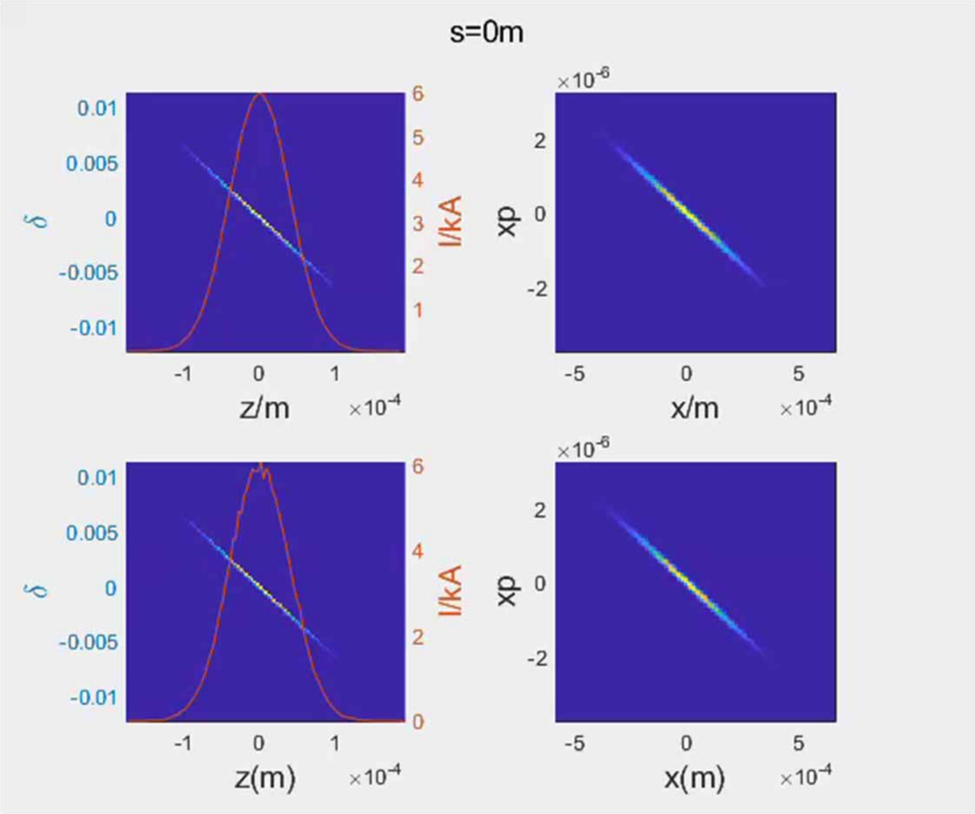
σ_z = 0.5 μm

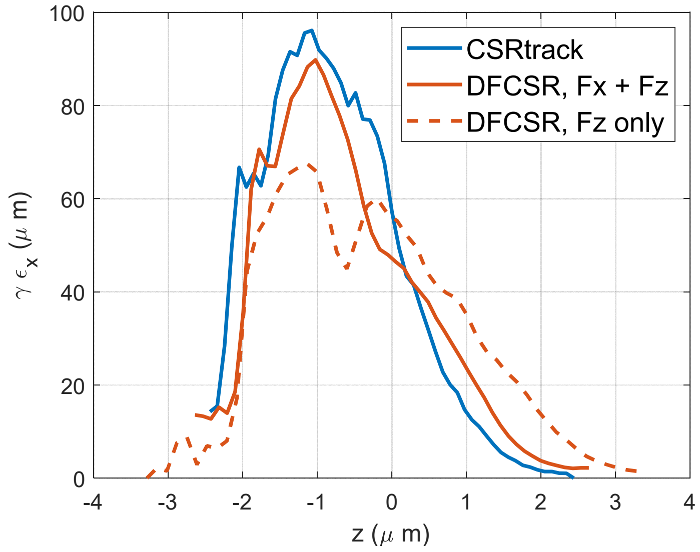


FACET-II Benchmark Chicane

DFCSR

CSRtrack





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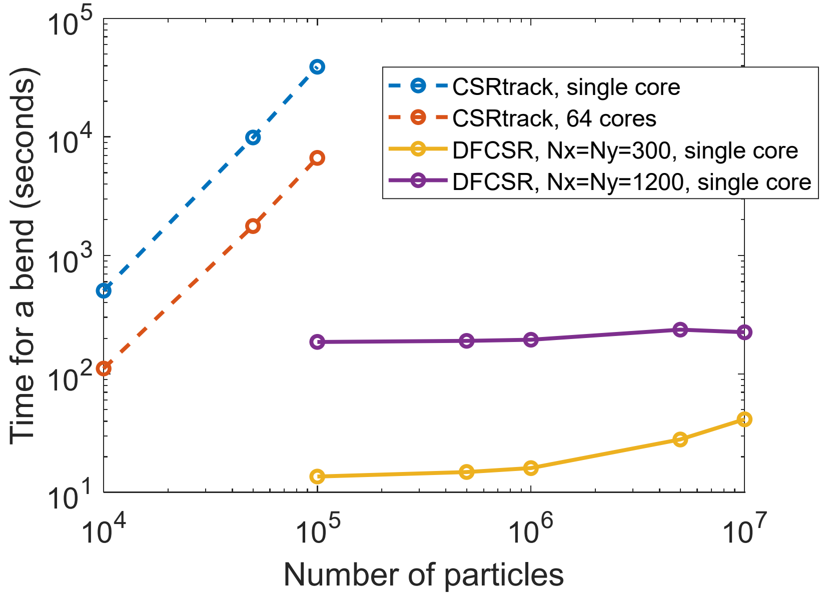
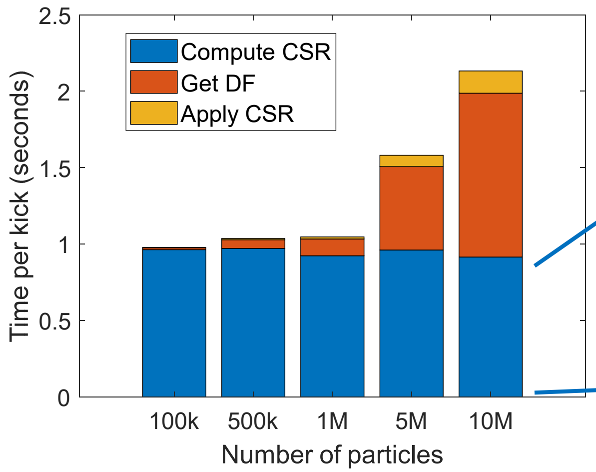
Computational Efficiency

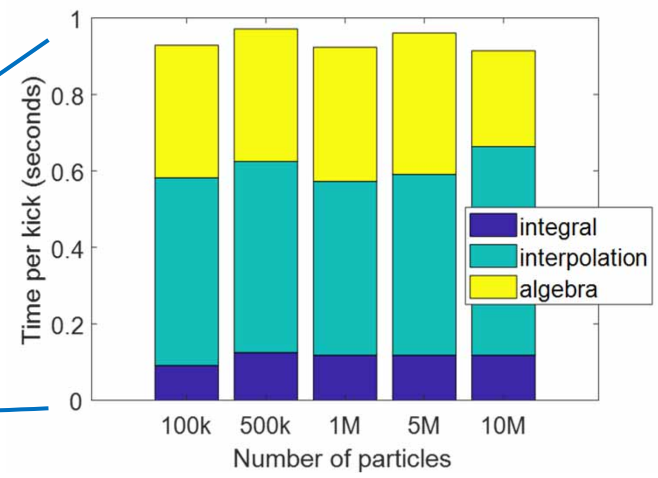


$$T = N_{part} \times \left(t_{deposit} + t_{apply\ field}\right)$$
$$+ N_x N_y N_{mesh} \times \left(t_{interp} + t_{integral}\right)$$
$$\sim O\left(N_{part}\right) + O\left(N_x N_y N_{mesh}\right)$$

$Memory\ estimate$:

$$N_{hist}^2 \times N_{steps} \sim 200^2 \times 100 \sim 32MB$$





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Open-sourced, user-friendly codes

- Open-sourced codes available both in Python and Matlab

  https://github.com/jy-tang/pyDFCSR

- Input defined by human-readable yaml files

input/init_beam.yaml

```yaml
n_particle: 2000000
px_dist:
  sigma_px:
    units: keV/c
    value: 22.2636
  type: gaussian
```

read from file or distgen

input/lattice.yaml

```yaml
element_1:
  type: drift
  L: 0.5
  nsep: 1
element_2:
  type: dipole
  L: 1.0
  angle: 1.0
  E1: 0
  E2: 0
  nsep: 1
```

- Single core or MPI parallel

Run DFCSR

```python
[2]: from CSR import *
     testCSR = CSR2D(input_file= 'input/chicane_config.yaml')
[3]: testCSR.run()
```

Parallel Run with MPI

```
# input yaml file in "./input/chicane_config.yaml"
!mpirun -n 4 python  -m pyDFCSR_mpi_run ./input/chicane_config.yaml
```

Open-sourced, user-friendly codes

- Outputs in self-described hdf5 files

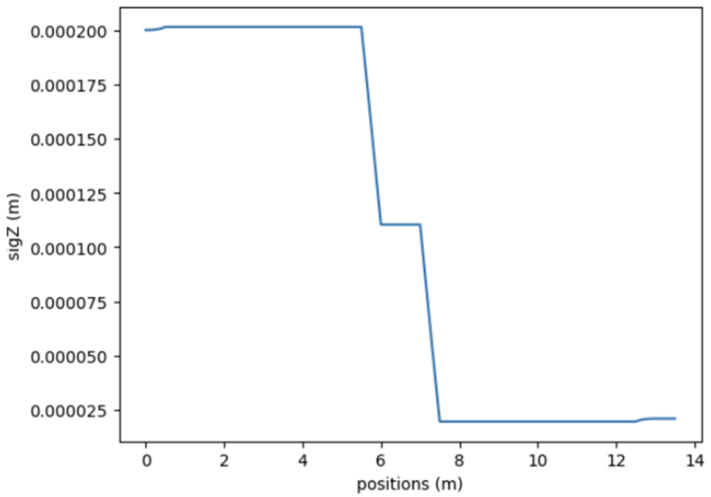
https://github.com/jy-tang/pyDFCSR/blob/main/pyDFCSR_2D/example.ipynb

Plot output

```
from plot_from_output import DFCSR_postprocessor

DFplot = DFCSR_postprocessor(run_name = 'chicane-2023-05-04T09_52_31-07_00',
                             work_dir = './output')
```

```
# Plot beam size along the lattice
DFplot.plot_stats(key = 'sigZ')
```

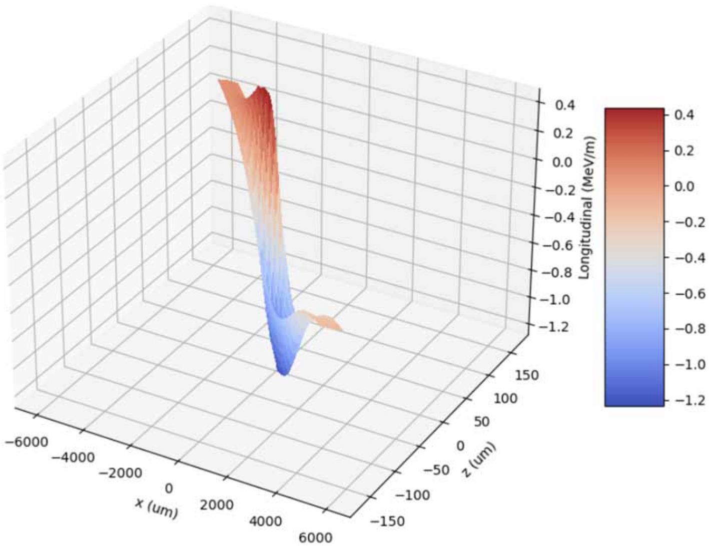


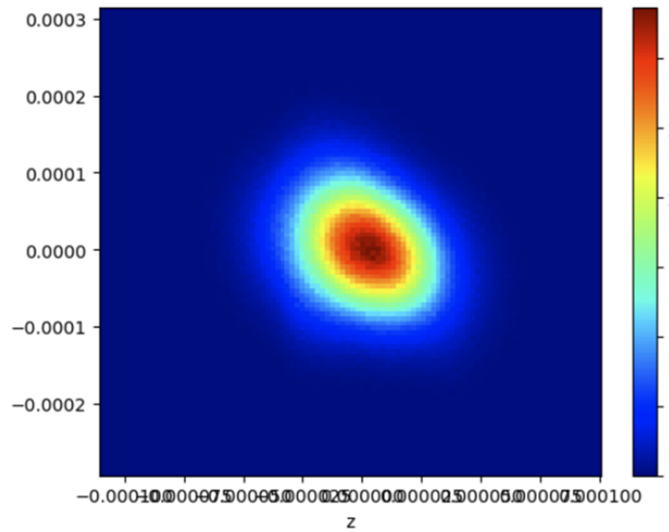

```
In [9]:  DFplot.plot_wakes(s = 7.25)
```

```
plot longitudinal wakes at nearest point s  = 7.29999999999999 m, step count 73
ebeam energy 5000000026.255665
```

```
In [12]:  DFplot.plot_particles(s = 12.75, xkey = 'z', ykey = 'x')
```

```
plot longitudinal wakes at nearest point s  = 12.7999999999997 m, step count 128
```





Thanks for your attention!

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