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LARMOR RADIUS EFFECT ON IFEL ACCELERATOR WITH STAGGERED UNDULATOR

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Abstract

In this paper, the theory of inverse free electron (IFEL) accelerator using staggered undulator has been discussed. The important contribution of staggered undulator parameter and the finite Larmor radius effect on energy saturation, saturation length and accelerating gradient of the IFEL accelerator are included in the analysis. Considering the synchrotron radiation losses, the IFEL accelerator equations are derived.

INTRODUCTION

The inverse free electron laser (IFEL) accelerator is the most advance laser based accelerator scheme [1-6]. It has been demonstrated that the accelerating gradient is significantly larger than what we achieve with conventional IF accelerator [7-9]. In the IFEL accelerator the energy transfer is from the laser beam to electrons. In an IFEL relativistic particles are moving through an undulator magnetic field with an electromagnetic wave propagating parallel to the beam. The undulator magnets produces a wiggling motion in a direction parallel to the electric vector of the laser. Hence energy is transferred from wave to the particle, if the resonance condition $\lambda = \frac{\lambda_u}{2\gamma^2}(1+K^2)$ satisfies where λ is the laser wavelength, λ_u is undulator wavelength and $K = \frac{eB_0\lambda_u}{2\pi mc^2}$ is undulator parameter. B_0 is the undulator magnetic field amplitude, '*m*' is the mass of particle, '*c*' speed of light. As the particle gain energy, the resonance condition of the free electron laser cannot be maintained for a long distance. The resonance condition can be maintained in two ways, firstly by changing the undulator period and secondly by changing the magnetic field of the undulator. The spectral properties of undulator radiation and the free electron laser gain in staggered undulator with the finite Larmor radius effect of electron are shown [10].

In this paper, we analyse the effect of Larmor radius on Inverse free electron laser accelerator with staggered undulator. We have obtained accelerator equation for inverse free electron laser accelerator. Synchrotron radiation losses are also included in the analysis. We have analysed the effect of Larmor radius on maximum energy gained by the electron and saturation length of the accelerator. Result and discussion of the analysis are given in the last section.

IFEL ACCELERATOR EQUATIONS

The motion and the energy change of relativistic electron in the presence of staggered array undulator and uniform magnetic field produced by a solenoid is calculated. The equation describing the motion of the electrons in the IFEL can be derived from the Lorentz equation of motion,

$$\frac{d\vec{\beta}}{dt} = \frac{e}{\gamma m_e c}\left[\vec{E}_L + \vec{\beta}\times\left(\vec{B}_L + \vec{B}_s\right)\right] \tag{1}$$

where B_L is the magnetic field of the laser and B_S is the magnetic field of the staggered undulator. e, m_e are the particle charge and mass respectively, c is the speed of light, $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = v/c$. We consider the magnetic field of a staggered array undulator with an axial magnetic field produced by a solenoid as

$$\vec{B}_s = [0, B_u\left(\sin\pi f/\pi f\right)\sin k_u z, B_0] \tag{2}$$

where λ_u is the undulator period with $k_u = 2\pi/\lambda_u$, $\omega_u = ck_u$, $f = \alpha/\lambda_u$, $\alpha = \lambda_u - d$ where '*d*' is the width of the rectangular pole and 'α' is the pole to pole gap .The undulator field strength is derived from a solenoid and reads

$$B_u = 2B_o/\sinh\left(\pi g/\lambda_u\right)$$

where 'B_0' is the axial field strength derived from the solenoid and '*g*' is the undulator gap. The electromagnetic wave propagating along the undulator is described by,

$$\vec{E}_L = \left[E_0\sin\psi, E_0\cos\psi, 0\right]$$
$$\vec{B}_L = \left[E_0\cos\psi, E_0\sin\psi, 0\right] \tag{3}$$

where $\psi = n(kz - \omega t)$ and $k = 2\pi/\lambda$, λ is wavelength of the laser. Using Eq. (1-3) for a staggered array undulator with an axial magnetic field, the electron velocity is given by,

$$\beta_x = \frac{\tilde{K}}{\gamma}\cos(\omega_u t) - \frac{K_L}{\gamma}\cos\psi + \beta_\perp\cos(\omega_u t) \tag{4a}$$

$$\beta_y = \beta_\perp\sin(\omega_u t) \tag{4b}$$

where the undulator and electromagnetic wave parameter is defined through

$$K = \frac{eB_u\lambda_u}{2\pi m_e c^2},\ K_L = \frac{eE_L\lambda}{2\pi m_e c^2},\ \tilde{K} = K\sin(\pi f)/\pi f$$

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Substituting β_x and β_y from equations (4a) and (4b) into the relation $\beta^2 = 1 - \frac{1}{\gamma^2}$, we obtain the longitudinal velocity as,

$$\beta_z = \beta^* - \frac{\tilde{K}^2}{4\gamma^2}\cos 2(\omega_u t) - \frac{K_L^2}{4\gamma^2}\cos 2\psi \tag{5}$$

Where

$$\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{\tilde{K}^2}{2} + \frac{K_L^2}{2} + \gamma^2\beta_\perp^2\right]$$

Using Eqs. (4) and (5), the change in electron energy is given by,

$$\frac{d\gamma}{dz} = A\frac{\tilde{K}}{2\gamma}\sin(\xi)\sum_m\left[J_{m-1}(0,\chi) - J_{m+1}(0,\chi)\right] \tag{6}$$

where

$$\chi = -\frac{n\tilde{K}^2 k}{8\gamma^2 k_u}$$

and phase term is

$$\xi = (n\,k + mk_u)\bar{z} - n\omega t \tag{7}$$

$J_{m-1}(0,\chi), J_{m+1}(0,\chi)$ are generalized Bessel functions of first kind.

Using the properties of generalized Bessel function and for $m = 1$, Eq (9) reduced to

$$\frac{d\gamma}{dz} = \tilde{A}\frac{\tilde{K}}{2\gamma}\left[J_0(\chi') - J_2(\chi')\right] \tag{8}$$

where $\tilde{A} = A\sin\xi$ and $\chi' = -\chi$

The resonance condition is read from Eq (7) by setting $d\xi / dz = 0$ as

$$\omega = \frac{2\gamma^2(\omega_u + \omega_c)}{\left[1 + \frac{\tilde{K}^2}{2} + \gamma^2\beta_\perp^2\right]} \tag{9}$$

For the electron trajectory such as given in Eq. (4) we get the synchrotron radiation loss term as,

$$\frac{1}{m_e c^2}\frac{dP}{dt} = \frac{1}{3}\frac{r_e}{c}\gamma^2\left[\omega_u^2\tilde{K}^2 + 2\beta_\perp^2\gamma^2\omega_c^2\right] \tag{10}$$

where $r_e = \frac{e^2}{m_e c^2}$ is the classical electron radius. Adding the radiation loss term and using resonance condition Eq (8) can be rewritten as,

$$\frac{d\gamma}{dz} = \tilde{A}\sqrt{\frac{\omega_u + \omega_c}{\omega}}[JJ] - \frac{2}{3}\frac{r_e}{c^2}\gamma^4\left[2\omega_u^2\frac{\omega_u + \omega_c}{\omega} + \beta_\perp^2\omega_c^2\right] \tag{11}$$

where $[JJ] = J_0(\chi') - J_2(\chi')$.

The solution for Eq. (11) is written as,

$$\gamma(z) = \gamma_\infty - [\gamma_\infty - \gamma_0]e^{-\frac{4F}{\gamma_\infty}(z-z_0)} \tag{12}$$

where γ_0 is the initial energy of the electron,

$$F = \tilde{A}\sqrt{\frac{\omega_u + \omega_c}{\omega}}[JJ]$$

and

$$\gamma_\infty = \left[\frac{3\tilde{A}c^2\sqrt{\frac{\omega_u + \omega_c}{\omega}}[JJ]}{2r_e\left[2\omega_u^2\frac{\omega_u + \omega_c}{\omega} + \beta_\perp^2\omega_c^2\right]}\right]^{1/4}$$

RESULTS AND DISCUSSION

We have analysed the staggered undulator with an axial magnetic field produced by a solenoid. The effect of synchrotron radiation losses derived in Eq.10 are included in the analysis. The relativistic Lorentz force equation is solved analytically for an electron beam having an initial finite perpendicular velocity. The electron entering the axial field executes Larmor motion. The Larmor motion is described by its Larmor radius defined as $r_L = c\beta_\perp / \omega_c$, where $c\beta_\perp$ is the initial transverse velocity and ω_c is the axial magnetic field strength. The effect of staggered undulator parameter and axial field on the performance of the IFEL accelerator is expressed in Eq. (12).

Figure 1 plots the accelerated energy for a 400 Giga Watt laser with 10.6 µm wavelength focused to a 240 µm spot size this gives an electric field strength of E=1.33×10^{6}statvolt cm^{-1}. Considering $r_e = 2.8179\times10^{-13}$ cm, $\tilde{A} = 667.686$ cm, λ_u=*50 mm*, $\alpha = 1$ and $r_L = 300$ mm in Fig. 1. We calculate energy in GeV for different value of $\beta_\perp$ ranges from 0 to 0.1. As the value of $\beta_\perp$ increases the maximum energy gained by the electron decreases.

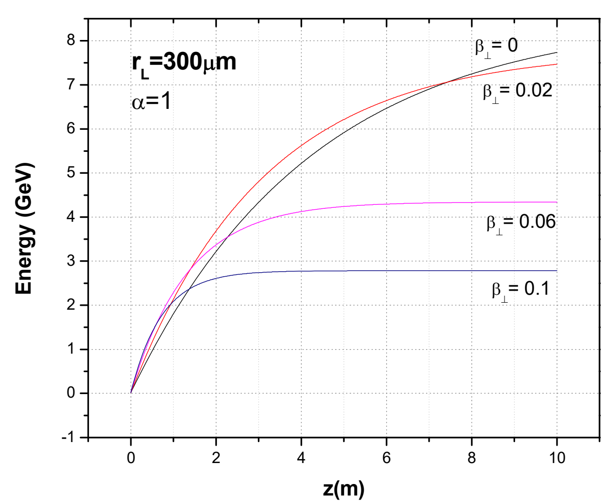

Figure 1: Electron energy vs distance for a fixed undulator wavelength.

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The synchrotron radiation loss term decides the length of the IFEL accelerator. In Fig. 2 we plot the accelerating gradient in GeV/m vs $\beta_\perp$ for two different values of pole to pole gap 'α'. The accelerating gradient decreases as $\beta_\perp$ increases from 0 to 0.3. The fall in accelerating gradient is 76.8% for $\alpha = 1$ and 83.28% for $\alpha = 4$.

In Fig. 3 we describe the variation of saturation length in m vs $\beta_\perp$. The saturation length decreases with $\beta_\perp$ ranges from 0 to 0.3. For $\alpha = 1$ saturation length decreases by 95.14% and for $\alpha = 4$ saturation length decreases by 95.26%.

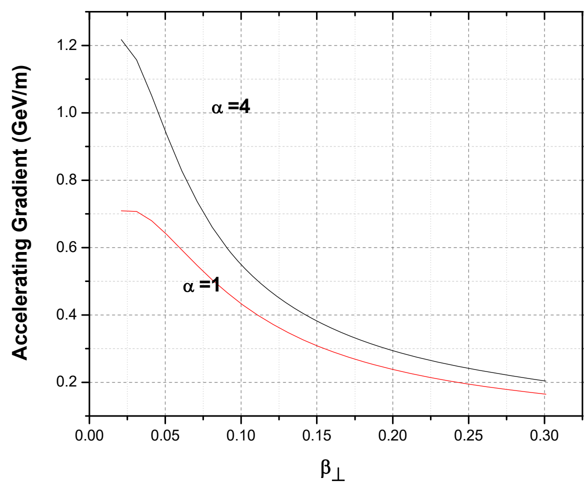

Figure 2: Accelerating gradient vs $\beta_\perp$.

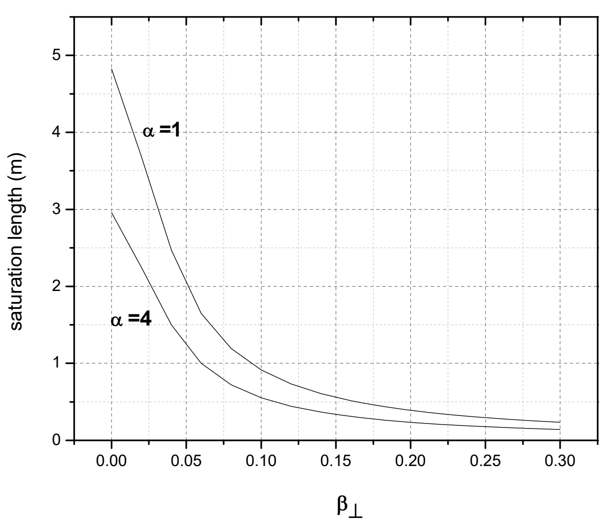

Figure 3: Saturation length vs $\beta_\perp$.

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