MULTI-FELOs DRIVEN BY A COMMON ELECTRON BEAM

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Abstract

Generating a free-electron laser (FEL) requires a high-brightness electron beam. To produce multiple FELs, the linac beam must be shared to enable one beam driving an undulator. This leads to a reduced average current and compromised FEL performance. Recently, a concept of multiple FELs driven by one electron beam was proposed, which enables reduction of equipment and improvement of productivity. We present here a simulation study based on an extended 1-D FEL oscillator model to demonstrate this concept. The system consists of two FEL oscillators arranged side-by-side and one electron beam passing through them. As such, the second, downstream oscillator is driven by bunches already been used once, while the first oscillator always receives fresh bunches from the linac. The study shows lasing could be achieved for both oscillators, their radiation intensities at saturation are comparable, thus meet needs of users. The concept also enables a potential application using a circulator ring such that an oscillator can be driven alternately by fresh linac bunches and from used bunches in the circulator ring. Extending the concept to cases of more than two FEL oscillators driven by one beam is also explored.

CONCEPT OF THE SCHEME

In 1984 Colella and Lucio [1] proposed the feasibility of a low-gain FELO emitting at 2 ∼ 3 Å using silicon single crystal diffraction around a right angle. With significant advance in accelerator technologies over the past 40 years, an electron beam with multi-GeV energy, multi-pC bunch charge, lower than 1 µm of the normalized emittance, and smaller than 10^{-4} of the energy spread can now be produced. In 2008 K.-J. Kim et al. [2] re-evaluated the idea and proposed using an FEL oscillator mode based on a high-repetition rate ERL-SRF and GeV electron beam to produce coherent radiation at 1 Å. Currently, the FEL community is actively investing in the development of x-ray FELO construction.

XFELO is a small-gain FEL, its undulator is much shorter compared to those used for SASE-like single-pass FELs, thus the radiation fields are built up slowly through many rounds in the optical cavity. Therefore, it is expected that the backreactions, namely interaction of the radiation fields on electrons, are usually much weaker, at least before the radiation fields reach saturation, and likely so after. As a consequence, perturbation on electrons, i.e., distortion of bunch phase space distribution, over each pass of individual short undulator is relatively small. This fact suggests that these lightly used bunches might be undegraded enough for driving next XFELOs in serial [3]. With proper design beam parameters, the electron bunches could be reused multiple times before being extracted and sent to beam dump. There will be a small reduction of gain for used bunches due to backreactions, but that could be compensated by lower losses in the optical cavity and it may also take more rounds of the light pulse to grow in the optical cavity. Eventually the radiation fields will reach a similar level of laser intensity at saturation, since the latter is determined by a balance of gain in undulator and combined loss of reflective crystal mirrors and radiation outcoupling.

MODEL DESCRIPTION

In this paper, we use a simple two-XFELO system to illustrate the concept, similar to the model used in Ref. [4]. The FELOs are arranged side-by-side as shown in Fig. 1. The driving electron beam from the source and SRF linac passes through undulators of the two XFELOs while interacting with and amplifying the radiation fields in both undulators. The spent electron bunch is extracted and goes to a beam dumper. At meantime, the next fresh bunch from the SRF linac enters the dual undulator system.

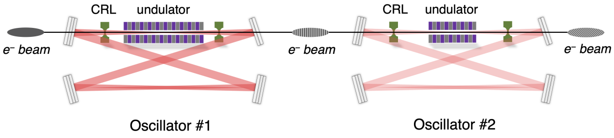

Figure 1: Conceptual schematic layout for the two-oscillator scheme. Bragg mirrors, consisting of three high-reflectivity mirrors and one mirror used for outcoupling, and two Be compound refractive lenses (CRLs) used for focusing are employed to form the oscillator. This design allows two independent users to conduct experiments.

The overall performance of the two x-ray FEL oscillators in serial is analyzed using an extended 1-D FEL model

$$\frac{d\theta_j}{d\tau} = p_j, \quad \frac{dp_j}{d\tau} = -\left(Ae^{i\theta_j} + c.c.\right), \quad \frac{dA}{d\tau} = \left\langle e^{-i\theta_j}\right\rangle + i\delta A \tag{1}$$

where τ is the scaled time along the undulator $\tau = 2\omega_u \rho t$, $\omega_u = c\beta_z k_u$, β_z is the longitudinal electron velocity along the undulator, $k_u = 2\pi/\lambda_u$ with λ_u the undulator period. The Pierce parameter $\rho = \left[\frac{1}{\gamma_r^3}\frac{I_b}{I_A}\frac{K^2[JJ]^2}{32\pi}\frac{\lambda_u^2}{2\pi\sigma_\perp^2}\right]^{\frac{1}{3}}$ with $[JJ] = J_0(\chi) - J_1(\chi), \chi = \frac{K^2}{4+2K^2}, K \approx 0.934\lambda_u[\mathrm{cm}]B_u[\mathrm{T}]$, $J_{0,1}$ being the 0th and 1st order of Bessel functions of the first kind. Here (θ_j, p_j) for $j = 1, 2, ..., N_e$ are the j-th electron phase space coordinate, with $p_j = \frac{\gamma_j - \gamma_r}{\rho\gamma_r}$. γ_r is the resonant electron energy (in unit of electron rest mass energy) satisfying the resonance condition $\lambda_r = \frac{\lambda_u}{2\gamma_r^2}\left(1 + \frac{K^2}{2}\right)$, $\omega_r = ck_r$,

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$k_r = 2\pi/\lambda_r$. $A = |A|e^{i\phi}$ is the complex scaled slow-varying radiation field amplitude with $|A|^2 = \frac{\epsilon_0|E|^2}{\rho n_e \gamma_r mc^2}$. The energy detuning parameter $\delta \equiv \frac{\langle\gamma_j\rangle_0 - \gamma_r}{\rho\gamma_r}$ where $\langle\gamma_j\rangle_0 = \langle\gamma_j\rangle(\tau = 0)$ and $b = \langle e^{-i\theta_j}\rangle$ is the bunching factor.

On each pass, within the undulator the electrons and radiation field evolve according to the above set of equations. To quantify the gain process inside the undulator, we define the following *gain parameter* [5] as $G \equiv 4\pi\rho N_u$. Note that this parameter is *not exactly* equivalent to the more familiar *intensity gain*, defined as $\mathcal{G} \equiv \left(|A_G|^2 - |A_0|^2\right)/|A_0|^2$, with $A_G = A(\tau = G)$ and $A_0 = A(\tau = 0)$. The radiation power outside the cavity, with the outcoupling coefficient α, can be related to the electron beam power as $P_{\text{out}} = \rho|A|^2 P_b \frac{\alpha}{\mathcal{L}}$, with $\mathcal{L} = \alpha + (1 - R)$ the total loss and α the outcoupling ratio [6]. To simulate two oscillators connected in serial, we extend the 1-D FEL model as follows. The subscript 1 is denoted to the quantities of the first oscillator and the subscript 2 to those of the second downstream oscillator. First, the electron beam will leave the first oscillator and serve the second one after transport $\theta_j \longrightarrow \theta_j + kR_{56}\rho_1 p_j$, with $kR_{56} = 2\pi R_{56}/\lambda_r$ and R_{56} being the momentum compaction factor between the oscillators. For simplicity here we assume the in-between section is isochronous ($kR_{56} = 0$). The radiation fields will bounce back and forth inside the cavity. In the 1-D model, the first-oscillator radiation field on next pass is updated by $A_{1,0}^{(n+1)} = r_1 A_{1,G}^{(n)}$ with $r_1 = \sqrt{R_1}$, assuming the reflectivity coefficient is real, same applies for the second oscillator. The superscript (n) means the n-th pass. For $n = 0$, the initial shot noise is given by $A_0 = \sqrt{\frac{6\sqrt{\pi}\rho}{N_\lambda\sqrt{\ln\frac{N_\lambda}{\rho}}}}$ with $N_\lambda = \frac{I_{\text{peak}}\lambda_r}{ce}$ and I_{peak} the peak current [5].

Because the second oscillator adopts the used electron beam, we have to include the effects due to beam quality degradation. While 3-D FEL model is much more complicated, there is one way to approximately take into account the effects of beam transverse finite emittance ϵ_{nx} and energy spread $\sigma_{\delta 0}$, so that we can simply replace ρ by ρ_{eff} [6], i.e., $\rho_{\text{eff}} = \rho\left(F_{\text{inh}}\, F_f\right)^{\frac{1}{3}}$, with $F_{\text{inh}} = \frac{1}{\left(1+1.7\mu_\gamma^2\right)\left(1+\mu_\epsilon^2\right)}$, $\mu_\gamma = 4N_u\sigma_\delta, \mu_\epsilon = \frac{\sqrt{2}N_u\epsilon_{nx}K}{\lambda_u\left(1+\frac{K^2}{2}\right)}$ and $F_f = \frac{1}{1+\left(\frac{\bar{w}}{2\sigma_b}\right)^2}$. Here $\sigma_b \approx \sqrt{\beta_{x,\text{ave}}\frac{\epsilon_{nx}}{\gamma_0}}$, using $\beta_{\text{x,ave}}^{\text{opt}} \approx \frac{L_u}{2\pi}$. $\bar{w}$ is the mean optical mode size, assuming $\bar{w} = \sqrt{2}\sigma_b$. When considering the effect of energy spread on the second oscillator, $\mu_\gamma \leq 1$. The effect of transverse emittance is relatively small ($\mu_\epsilon \approx 10^{-3}$) and can be neglected here.

When the electron beam enters the second oscillator, we should re-normalize the electron beam parameters according to the design of the second oscillator, i.e., $p_j\left(\tau_2 = 0\right) = \left(\frac{\rho_{\text{eff},1}}{\rho_{\text{eff},2}}\right) p_j\left(\tau_1 = G_1\right)$.

The Pierce parameter of the second oscillator should be updated every pass, because the input beam properties may change significantly before the first oscillator reaches an equilibrium. This update $\rho_2 \longrightarrow \rho_{\text{eff},2}$ is necessary in order to take into account the change of beam properties, especially the beam energy spread. The Pierce parameter reflects how effective the electron-radiation interaction will be; that is, $\rho_{\text{eff},2}$ is a function of both the second oscillator and of the first oscillator. The insightful idea of using a common electron beam to drive multiple x-ray FELOs really appreciates the re-use of Pierce parameter. When the FELO reaches equilibrium for each oscillator, the intensity gain $\mathcal{G}$ and loss are balanced, and the radiation field in the oscillator no longer increases or decreases when the electron beam passes through it. For simplicity, the average net energy loss of the electron beam in the first oscillator is ignored here, assuming it can be compensated by proper detuning in the second oscillator. We use the standard numerical methods to solve Eqs. (1). Here we note that the above 1-D FELO oscillator model makes the steady-state approximation. Time-dependent and 3-D effects will inevitably degrade the design performance.

Table 1: Electron Beam and Undulator Parameters

Name	Value	Unit
Resonant electron energy E	5.7	GeV
Bunch length, rms σ_t	0.55	ps
Bunch charge Q	380	pC
Bunch current I_b	300	A
Normalized emittance ϵ_{nx}	0.3	μm
Energy spread σ_δ	0.3×10^{-4}	
Undulator period λ_u	1.88	cm
No. of undulator periods N_{u1}, N_{u2}	100/68	
Undulator parameter K	1.5	
Resonant wavelength λ_r	1.6	Å
Round-trip reflectivity R	0.8	
Outcoupling ratio α	0.04	

MAIN RESULTS

We use a set of preliminary design to demonstrate the feasibility of the proposed scheme. The linac beam and undulator parameters are summarized in Table 1. Because the number of undulator periods in our design is relatively small, a pulse length of 0.55 ps is required to ensure that the bandwidth of the undulator radiation generated per pass is narrow enough to fall within the range of the Bragg crystal mirror's reflectivity curve. Assuming a crystal reflectivity width of approximately 10^{-6}, a pulse length of 0.55 ps can accommodate approximately 10^6 wavelengths. Therefore, the rms pulse length of 0.55 ps can generate undulator radiation with a bandwidth sufficient to meet the requirements.

Despite the shorter undulator, we utilize the high bunch charge of the CEBAF injector and a higher current to achieve a proper single-pass gain G, which is approximately 1.4 for the first oscillator and 0.8 for the second oscillator. The value of G should be chosen to be moderate, not too small to compensate for round-trip losses, but also avoiding being too large, which could cause the electron beam to undergo synchrotron oscillations in the undulator and saturate too early. For the second oscillator, it is recommended to appropriately reduce the number of undulator periods because the driving electron beam is already microbunched. The total reflectivity (or round-trip reflectivity) is approximately $0.96^3 \times 0.997^2 \times 0.92 = 0.8$ for the two XFELOs.

Due to the operation of the oscillator in the low-gain regime, the initial energy offset of p_0 depends on the initial detune δ and the resonant energy. For the first oscillator, we apply the well-known result of $p_0 = \delta \approx 2.6/G_1$, where $G_1 = 4\pi\rho_1 N_{u1}$. For the second oscillator, although the electron beam may have been a bit modulated, we still choose $p_0 = \delta \approx 2.6/G_2$, where $G_2 = 4\pi\rho_2 N_{u2}$, as an acceptable choice for a preliminary feasibility demonstration. We calculate the rms energy spread before entering the second oscillator each turn and then update the Pierce parameter of the second oscillator each turn.

Figure 2 presents the numerical results based on Table 1. The output power of the first FELO is consistent with the rough estimate formula $P_{\text{sat}} \approx \frac{\alpha P_{\text{beam}}}{N_u(1-R)} \approx 3.4$ GW. In this example, the gain parameters $G_{1,2}$ of the two oscillators are 1.42 and 0.96, respectively, corresponding to the initial Pierce parameters of approximately 1.126×10^{-3}. The ideal detune is chosen the way just mentioned. We note that the choice of G is recommended to be between 1 and 1.5. If it is too small, the efficiency may be poor; if it is too large, the undulator may quickly saturate. For simplicity, we assume that the in-between transport section between the two FELOs is isochronous, i.e., $kR_{56} = 0$.

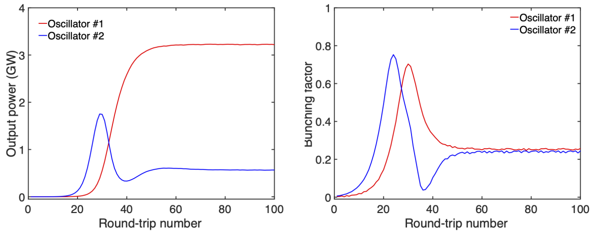

Figure 2: Evolution of output power (left) and bunching factor (right) for the two oscillators.

In the early stage of the first oscillator reaching saturation, the radiation field amplitude in the oscillator starts to accumulate with each passing round, modulating the electron beam. Note that the electron beam is modulated by the radiation field before the radiation field amplitude is enhanced. As seen in Fig. 2, the bunching factor on the right panel grows earlier than the radiation field amplitude on the left. Moreover, for the almost (but not exactly) optimal electron beam modulation in the first oscillator, it reaches the optimum bunching factor after passing through a certain distance of the undulator in the second oscillator. This explains why the peak of the blue line in Fig. 2(b) always occurs before the peak of the red line.

Figure 3 displays the evolution of the energy spread at the entrance of the second oscillator and the modified Pierce parameter $\rho_{\text{eff},2}$ of the second oscillator. It can be observed that the energy spread of the electron beam inevitably increases after driving the first oscillator, leading to a decrease in $\rho_{\text{eff},2}$. It has been conventionally believed that the excitation efficiency would be compromised when the energy spread exceeds the Pierce parameter. However, when the first oscillator saturates, the disturbance on the electron beam is relieved, and most of the electrons located at the bottom of the potential well can still be effectively modulated in the second oscillator, despite the overall energy spread exceeding the Pierce parameter. Since only a fraction of the electrons in the second oscillator participate in lasing, the output power is relatively lower, but still appreciable.

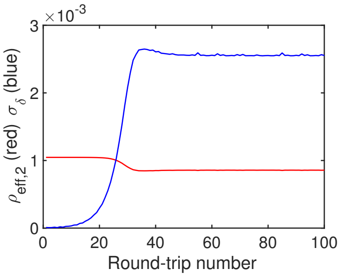

Figure 3: Beam energy spread and Pierce parameter at the entrance of the second oscillator.

SUMMARY AND DISCUSSION

We have investigated the feasibility of using the same electron beam to drive two XFELOs by extending the 1-D FEL analysis, which builds on the previous work of driving a single XFELO. The proposed scheme assumes a high repetition rate, high brightness electron beam with the energy of 5.7 GeV and the peak current of 300 A, to drive two fully coherent hard x-ray lasers, each with the light pulse length comparable to the electron bunch duration 0.5 ps. The two x-ray lasers have output powers of approximately 3.2 GW and 0.6 GW, peak brightnesses of 2×10^{32} and 0.4×10^{32} photons/sec/(mm mr)^2×(0.1% BW), and average brightnesses of 10^{27} and 2×10^{26} photons/sec/(mm mr)^2×(0.1% BW), respectively, assuming a repetition rate of 0.462 MHz [3]. This proposal has practical value, as it reduces the cost for both the facility and each user to some extent, while providing peak brightness comparable to that of the currently under construction LCLS-II, and average brightness at the same photon energy comparable or even higher by about an order of magnitude than the planned LCLS-II-HE [7]. The concept also enables a potential application using a circulator ring such that an oscillator can be driven alternately by fresh linac bunches and from used bunches in the circulator ring. In such operation, the high-brightness electron beam traversing multiple bends may induce the coherent synchrotron radiation (CSR) effect [8, 9] or other possibly undesired collective effects [10], which are beyond the scope of this analysis but will be considered in future studies. The present analysis is mostly based on numerical simulations. Further analytical [11] or semi-analytical analysis is necessary to better understand the performance of downstream oscillators in a serial FELO configuration.

ACKNOWLEDGMENTS

The authors wish to thank Dr. Weilun Qin (DESY) for helpful discussions. This manuscript has been authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. CT is supported by the Fundamental Research Funds for the Central Universities (HUST) under Project No. 2021GCRC006 and National Natural Science Foundation of China under project No. 12275094.

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