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OPTICAL STOCHASTIC COOLING IN A GENERAL COUPLED LATTICE

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Abstract

Here we present a formalism of optical stochastic cooling in a 3D general coupled lattice. The formalism is general, and can treat a variety of damping and diffusion mechanisms within a single framework. We expect the work to be of value for the development of future light source.

GENERAL FORMALISM OF STORAGE RING PHYSICS

Particle state vector $\mathbf{X} = (x, x', y, y', z, \delta)^T$ is used throughout this paper, with its components meaning the horizontal position, horizontal angle, vertical position, vertical angle, longitudinal position, and relative energy difference of a particle with respect to the reference particle, respectively. The superscript ^{T} means the transpose of a vector or matrix. Following Chao's solution by linear matrix (SLIM) formalism [1], we can introduce the definition of the generalized beta functions in a 3D general coupled storage ring lattice as

$$\beta_{ij}^k = 2\text{Re}\left(\mathbf{E}_{ki}\mathbf{E}_{kj}^*\right),\ k = I, II, III, \tag{1}$$

where * means complex conjugate, the sub or superscript k denotes one of the three eigenmodes, Re() means the real component of a complex number or matrix, $\mathbf{E}_{ki}$ is the i-th component of vector $\mathbf{E}_k$, and $\mathbf{E}_k$ are eigenvectors of the 6×6 symplectic one-turn map $\mathbf{M}$ with eigenvalues $e^{i2\pi\nu_k}$, satisfying the following normalization condition

$$\mathbf{E}_k^\dagger\mathbf{S}\mathbf{E}_k = \begin{cases} i,\ k = I, II, III, \\ -i,\ k = -I, -II, -III, \end{cases} \tag{2}$$

and $\mathbf{E}_k^\dagger\mathbf{S}\mathbf{E}_j = 0$ for $k \neq j$, where † means complex conjugate transpose, and

$$\mathbf{S} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}. \tag{3}$$

Since the one-turn map is a real symplectic matrix, for a stable motion, we have

$$\nu_{-k} = -\nu_k,\ \mathbf{E}_{-k} = \mathbf{E}_k^*. \tag{4}$$

Using the generalized beta function, we can write the eigenvector component as

$$\mathbf{E}_{kj} = \sqrt{\frac{\beta_{jj}^k}{2}}e^{i\phi_j^k}. \tag{5}$$

And according to definition we have

$$\beta_{ij}^k = \sqrt{\beta_{ii}^k\beta_{jj}^k}\cos(\phi_i^k - \phi_j^k). \tag{6}$$

Similarly, here we introduce the definition of imaginary generalized beta functions as

$$\hat{\beta}_{ij}^k = 2\text{Im}\left(\mathbf{E}_{ki}\mathbf{E}_{kj}^*\right),\ k = I, II, III, \tag{7}$$

where Im() means the imaginary component of a complex number or matrix. Further we can define the real and imaginary generalized Twiss matrices of a storage ring lattice corresponding to three eigen mode as

$$(\mathbf{T}_k)_{ij} = \beta_{ij}^k,\ \left(\hat{\mathbf{T}}_k\right)_{ij} = \hat{\beta}_{ij}^k,\ k = I, II, III. \tag{8}$$

Due to the symplecticity of the one-turn map, we have

$$\mathbf{T}_k^T = \mathbf{T}_k,\ \hat{\mathbf{T}}_k^T = -\hat{\mathbf{T}}_k, \tag{9}$$

where ^{T} means transpose. The generalized Twiss matrices at different places are related according to

$$\begin{aligned} \mathbf{T}_k(s_2) &= \mathbf{R}(s_2, s_1)\mathbf{T}_k(s_1)\mathbf{R}^T(s_2, s_1), \\ \hat{\mathbf{T}}_k(s_2) &= \mathbf{R}(s_2, s_1)\hat{\mathbf{T}}_k(s_1)\mathbf{R}^T(s_2, s_1), \end{aligned} \tag{10}$$

with $\mathbf{R}(s_2, s_1)$ being the transfer matrix from s_1 to s_2.

The action or generalized Courant-Snyder invariants of a particle are defined according to

$$J_k \equiv \frac{\mathbf{X}^T\mathbf{G}_k\mathbf{X}}{2},\ k = I, II, III, \tag{11}$$

where

$$\mathbf{G}_k \equiv \mathbf{S}^T\mathbf{T}_k\mathbf{S}. \tag{12}$$

It is easy to prove that J_k are invariants of a particle when it travels around the ring, from the symplectic condition $\mathbf{R}^T\mathbf{S}\mathbf{R} = \mathbf{S}$. The three eigenemittance of a beam containing N_p particles are defined according to

$$\epsilon_k \equiv \langle J_k\rangle = \frac{\sum_{i=1}^{N_p} J_{k,i}}{N_p},\ k = I, II, III, \tag{13}$$

where $J_{k,i}$ means the k-th mode invariant of the i-th particle.

Assume there is a perturbation $\mathbf{K}$ to the one-turn map $\mathbf{M}$, i.e., $\mathbf{M}_{\text{per}} = (\mathbf{I} + \mathbf{K})\mathbf{M}_{\text{unp}}$. From cannonical perturbation theory [2], the tune shift of the k-th eigen mode is then

$$\Delta\nu_k = -\frac{1}{4\pi}\text{Tr}\left[\left(\mathbf{T}_k + i\hat{\mathbf{T}}_k\right)\mathbf{S}\mathbf{K}\right], \tag{14}$$

where Tr() means the trace of a matrix. This formula can be used to calculate the real and imaginary tune shifts due to symplectic (for example lattice error) and non-symplectic

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(for example radiation damping) pertubations. The pertubation theory can also be applied to calcuate the emittance growth due to diffusion [2]. With the help of real and imaginary generalized beta functions and Twiss matrices, the diffusion of emittance per turn can be calculated as

$$\Delta\epsilon_k = -\frac{1}{2}\oint \mathrm{Tr}\left(\mathbf{T}_k\mathbf{SNS}\right) ds = \frac{1}{2}\oint \mathrm{Tr}\left(\mathbf{G}_k\mathbf{N}\right) ds, \quad (15)$$

and the damping rate of each eigen mode is

$$\alpha_k = -\frac{1}{2}\oint \mathrm{Tr}\left(\hat{\mathbf{T}}_k\mathbf{SD}\right) ds, \quad (16)$$

where $\mathbf{N}$ and $\mathbf{D}$ are the diffusion and damping matrix, respectively. Note that the damping rates here are that for the corresponding eigenvectors. The damping rates for particle action or beam emittance is a factor of two larger. The equilibrium eigenemittance between a balance of diffusion and damping can be calculated as

$$\epsilon_k = \frac{\Delta\epsilon_k}{2\alpha_k} = \frac{-\frac{1}{2}\sum_{i,j}\oint \beta^k_{ij}\left(\mathbf{SNS}\right)_{ij} ds}{\sum_{i,j}\oint \hat{\beta}^k_{ij}\left(\mathbf{SD}\right)_{ij} ds}, \quad (17)$$

After getting the equilibrium eigenemittances, the second moments of beam can be written as

$$\Sigma_{ij} = \sum_{k=I,II,III}\epsilon_k\beta^k_{ij}, \quad (18)$$

or in matrix form as

$$\mathbf{\Sigma} = \sum_{k=I,II,III}\epsilon_k\mathbf{T}_k. \quad (19)$$

QUANTUM EXCITATION AND RADIATION DAMPING

In an electron storage ring, the intrinsic diffusion and damping are both from the emission of photons, i.e., quantum excitation and radiation damping. For quantum excitation, we have all the other components of diffusion matrix $\mathbf{N}$ zero except that

$$N_{66} = \frac{2C_L\gamma^5}{c|\rho|^3} \quad (20)$$

where c is the speed of light in free space, ρ is the benging radius of particle trajectory, γ here is the relativistic factor, $C_L = \frac{55}{48\sqrt{3}}\frac{r_e\hbar}{m_e}$ with r_e the classical electron radius, $\hbar$ the reduced Planck's constant, m_e the electron mass.

For radiation damping, we have two sources of damping, i.e., dipole magnets and RF cavity. For a horizontal dipole, we have all the matrix terms of $\mathbf{D}$ zero except that

$$D_{66} = -\frac{1}{\pi}C_\gamma\frac{E_0^3}{\rho^2},\ D_{61} = -\frac{C_\gamma E_0^3}{2\pi}\frac{1-2n}{\rho^3}, \quad (21)$$

where $C_\gamma = \frac{4\pi}{3}\frac{r_e}{(m_ec^2)^3} = 8.85\times10^{-5}\frac{\mathrm{m}}{\mathrm{GeV}^2}$, $n = -\frac{\rho}{B_y}\frac{\partial B_y}{\partial x}$ is the transverse field gradient index. For an RF cavity, we have all the matrix terms of $\mathbf{D}$ zero except that

$$D_{22} = D_{44} = -\frac{U_0}{E_0}\delta(s_{\mathrm{RF}}), \quad (22)$$

where U_0 is the radiation energy loss of a particle per turn, E_0 is the particle energy, and $\delta(s)$ means Dirac's delta function. Here we have assumed that the RF cavity is a zero-length one. Using the developed formalism, it is easy to show that for radiation damping, we have

$$\alpha_I + \alpha_{II} + \alpha_{III} = -\frac{1}{2}\oint \mathrm{Tr}(\mathbf{D})ds = \frac{2U_0}{E_0}, \quad (23)$$

which is the well-known Robinson's sum rule [3].

In a planar uncoupled electron storage ring, this general formalism reduces to the classical results of Sands, i.e., the radiation integrals formalism found in textbooks [4]. More specifically in this case we have the equilibrium emittance

$$\begin{aligned}
\epsilon_x &= \frac{C_L\gamma^5}{2c\alpha_I}\oint\frac{\beta^I_{55}}{|\rho(s)|^3}ds = \frac{C_L\gamma^5}{2c\alpha_I}\oint\frac{\mathcal{H}_x(s)}{|\rho(s)|^3}ds,\\
\epsilon_y &= \frac{C_L\gamma^5}{2c\alpha_{II}}\oint\frac{\beta^{II}_{55}}{|\rho(s)|^3}ds = \frac{C_L\gamma^5}{2c\alpha_{II}}\oint\frac{\mathcal{H}_y(s)}{|\rho(s)|^3}ds, \quad (24)\\
\epsilon_z &= \frac{C_L\gamma^5}{2c\alpha_{III}}\oint\frac{\beta^{III}_{55}}{|\rho(s)|^3}ds = \frac{C_L\gamma^5}{2c\alpha_{III}}\oint\frac{\beta_z(s)}{|\rho(s)|^3}ds,
\end{aligned}$$

with the $\mathcal{H}$-function defined as $\mathcal{H} = \gamma D^2 + 2\alpha DD' + \beta D'^2$, where α, β, γ are the classical Courant-Snyder functions [5], and the damping constants, according to Eq. (16), being

$$\begin{aligned}
\alpha_I &= \frac{U_0}{2E_0}\left(1 - \frac{\oint D_x\left(\frac{1-2n}{\rho^3}\right)ds}{\oint\frac{1}{\rho^2}ds}\right),\\
\alpha_{II} &= \frac{U_0}{2E_0}, \quad (25)\\
\alpha_{III} &= \frac{U_0}{2E_0}\left(2 + \frac{\oint D_x\left(\frac{1-2n}{\rho^3}\right)ds}{\oint\frac{1}{\rho^2}ds}\right).
\end{aligned}$$

OPTICAL STOCHATIC COOLING

Damping Rate in Linear Approximation

Now we apply the formalism to optical stochastic cooling (OSC) [6–9]. Denote the symplectic transfer matrix of particle state evector from the pick-up undulator to the kiker undulator as $\mathbf{R}$. Assume that the change of a particle's energy induced in the kicker undulator due to its own radiation at the pick-up undulator is

$$\Delta\delta = -A\sin(k_R\Delta z) \quad (26)$$

with

$$\begin{aligned}
\Delta z(s_2,s_1) = &R_{51}x_1 + R_{52}x'_1 + R_{53}y_1 + R_{54}y'_1\\
&+ R_{55}z_1 + R_{56}\delta_1 - z_1,
\end{aligned} \quad (27)$$

where we have used the subscripts 1 and 2 to represent the location of pick-up undulator and kicker undulator, respectively. Usually we have $R_{55} = 1$.

Linearizing the energy kick around the zero-crossing phase, we have the effective change of state vector at the pick-up undulator as

$$\Delta\mathbf{X}_1 = \mathbf{D}\mathbf{X}_1, \quad (28)$$

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with the perturbation matrix at the pick-up undulator

$$\mathbf{D} = -Ak_R \begin{pmatrix} -R_{51}R_{52} & -R_{52}^2 & -R_{52}R_{53} & -R_{52}R_{54} & -(R_{55}-1)R_{52} & -R_{52}R_{56} \\ R_{51}^2 & R_{51}R_{52} & R_{51}R_{53} & R_{51}R_{54} & (R_{55}-1)R_{51} & R_{51}R_{56} \\ -R_{51}R_{54} & -R_{52}R_{54} & -R_{53}R_{54} & -R_{54}^2 & -(R_{55}-1)R_{54} & -R_{54}R_{56} \\ R_{51}R_{53} & R_{52}R_{53} & R_{53}^2 & R_{53}R_{54} & (R_{55}-1)R_{53} & R_{53}R_{56} \\ -R_{51}R_{56} & -R_{52}R_{56} & -R_{53}R_{56} & -R_{54}R_{56} & -(R_{55}-1)R_{56} & -R_{56}^2 \\ R_{51}R_{55} & R_{52}R_{55} & R_{53}R_{55} & R_{54}R_{55} & (R_{55}-1)R_{55} & R_{55}R_{56} \end{pmatrix}. \tag{29}$$

Then, we have the sum rule for the OSC damping rates of three eigen modes

$$\alpha_{I,0} + \alpha_{II,0} + \alpha_{III,0} = -\frac{1}{2}\mathrm{Tr}\,(\mathbf{D}) = \frac{Ak_R R_{56}}{2}. \tag{30}$$

The subscript 0 is used to denote that the damping rates are calculated by linearizing the energy kick around the zero-crossing phase. The OSC damping rate of each eigen mode can be calculated according to Eq. (16). More specifically,

$$\begin{aligned} \alpha_{I,0} &= -\frac{Ak_R}{2}\Big(R_{51}\hat{\beta}_{51}^{I} + R_{52}\hat{\beta}_{52}^{I} \\ &\qquad + R_{53}\hat{\beta}_{53}^{I} + R_{54}\hat{\beta}_{54}^{I} + R_{56}\hat{\beta}_{56}^{I}\Big), \\ \alpha_{II,0} &= -\frac{Ak_R}{2}\Big(R_{51}\hat{\beta}_{51}^{II} + R_{52}\hat{\beta}_{52}^{II} \\ &\qquad + R_{53}\hat{\beta}_{53}^{II} + R_{54}\hat{\beta}_{54}^{II} + R_{56}\hat{\beta}_{56}^{II}\Big), \\ \alpha_{III,0} &= -\frac{Ak_R}{2}\Big(R_{51}\hat{\beta}_{51}^{III} + R_{52}\hat{\beta}_{52}^{III} \\ &\qquad + R_{53}\hat{\beta}_{53}^{III} + R_{54}\hat{\beta}_{54}^{III} + R_{56}\hat{\beta}_{56}^{III}\Big). \end{aligned} \tag{31}$$

Amplitude-dependent Damping Rate

In the above analysis, we have linearized the sinusoidal energy kick around the zero-crossing phase. Without such approximation, the damping rates will be different for particles with different betatron or synchrotron amplitudes. The betatron and synchrotron oscillation-averaged damping rates in a 3D general coupled lattice are then

$$\begin{aligned} \alpha_I &= 2\alpha_{I,0}\frac{J_1(k_R a_I)J_0(k_R a_{II})J_0(k_R a_{III})}{k_R a_I}, \\ \alpha_{II} &= 2\alpha_{II,0}\frac{J_0(k_R a_I)J_1(k_R a_{II})J_0(k_R a_{III})}{k_R a_{II}}, \\ \alpha_{III} &= 2\alpha_{III,0}\frac{J_0(k_R a_I)J_0(k_R a_{II})J_1(k_R a_{III})}{k_R a_{III}}, \end{aligned} \tag{32}$$

with J_n the n-th order Bessel function of the first kind, and

$$\begin{aligned} a_I &= \sqrt{2J_I\left[\beta_{11}^{I}R_{51}^2 + 2\beta_{12}^{I}R_{51}R_{52} + \beta_{22}^{I}R_{52}^2\right]}, \\ a_{II} &= \sqrt{2J_{II}\left[\beta_{33}^{II}R_{53}^2 + 2\beta_{34}^{II}R_{53}R_{54} + \beta_{44}^{II}R_{54}^2\right]}, \\ a_{III} &= \sqrt{2J_{III}\left[\beta_{55}^{III}R_{55}^2 + 2\beta_{56}^{III}R_{55}R_{56} + \beta_{66}^{III}R_{56}^2\right]}, \end{aligned} \tag{33}$$

where $J_{I,II,III}$ mean the generalized Courant-Snyder invariants of the particle.

The first roots of $J_0(x)$ and $J_1(x)$ are $\mu_{01} \approx 2.405$ and $\mu_{11} \approx 3.83$. The range of betatron and synchrotron oscillation amplitude which gives a positive damping rate is called cooling range. If we want a cooling range a factor of N larger than RMS oscillation amplitude of the particle beam in all three modos, then we need $Nk_R\bar{a}_k < \mu_{01}$, $k = I, II, III$, where $\bar{a}_k$ is a_k with $2J_k$ in the expression replaced by ϵ_k. For example, $\bar{a}_I = \sqrt{\epsilon_I\left[\beta_{11}^{I}R_{51}^2 + 2\beta_{12}^{I}R_{51}R_{52} + \beta_{22}^{I}R_{52}^2\right]}$. The physical meaning of $\bar{a}_k$ is the RMS lengthening of a longitudinal slice from pick-up to kicker undulator from the k-mode eigen emittance. If we only need cooling in one mode, then the cooling range can be larger, i.e., $Nk_R\bar{a}_k < \mu_{11}$.

Planar Uncoupled Ring

The above results apply for a 3D general coupled lattice. For a ring without x-y coupling and when RF cavity is placed at dispersion-free location, we can express the normalized eigenvectors using classical Courant-Snyder functions and dispersion D and dispersion angle D' [10]. Then we have

$$\begin{aligned} \alpha_{I,0} &= -\frac{Ak_R\left(R_{51}D_{x1} + R_{52}D'_{x1}\right)}{2}, \\ \alpha_{III,0} &= \frac{Ak_R R_{56}}{2} - \alpha_{I,0}, \end{aligned} \tag{34}$$

or in a more elegant form as

$$\begin{aligned} \alpha_{I,0} &= \frac{Ak_R}{2}\sqrt{\mathcal{H}_{x1}\mathcal{H}_{x2}}\sin\left(\Delta\psi_{x21} - \Delta\chi_{x21}\right), \\ \alpha_{III,0} &= \frac{Ak_R}{2}F, \end{aligned} \tag{35}$$

where $\Delta\psi_{x21} = \psi_{x2} - \psi_{x1} = \int_{s_1}^{s_2}\frac{1}{\beta_x}ds$ is the the horizontal betatron phase advance, and $\Delta\chi_{x21} = \chi_{x2} - \chi_{x1}$ is the horizontal chromatic phase advance, from the pick-up to kicker undulator, and

$$F(s_2, s_1) = -\int_{s_1}^{s_2}\left(\frac{D_x(s)}{\rho(s)} - \frac{1}{\gamma^2}\right)ds. \tag{36}$$

To obtain the final concise result, D and D' have been expressed in terms of the chromatic $\mathcal{H}$-function and the chromatic phase χ, according to

$$D = \sqrt{\mathcal{H}\beta}\cos\chi,\ D' = -\sqrt{\mathcal{H}/\beta}\left(\alpha\cos\chi + \sin\chi\right). \tag{37}$$

Some observations are in order based on Eq. (35). First, to induce damping on the eigen mode III, which usually corresponds to the longitudinal dimension, we need a nonzero F. Second, to induce damping on mode I, which usually corresponds to the horizontal dimension, both the pick-up and kicker undulators need to be placed at dispersive locations. Further, we need to make sure the chromatic phase advance between the two undulators is different from the corresponding betatron phase advance, and the sign of damping rate depends on the difference of chromatic and betatron phase advance. For example, if it is an achromat between pick-up and kicker undulators, which means $R_{51} = 0$ and $R_{52} = 0$, then there will be no damping on the eigen mode I.

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The amplitude-dependent damping rates in this case are

$$\alpha_I = -\frac{A\left(R_{51}D_{x1} + R_{52}D'_{x1}\right)}{\sqrt{2J_x\left[\beta_{x1}R_{51}^2 - 2\alpha_{x1}R_{51}R_{52} + \gamma_{x1}R_{52}^2\right]}} J_1\left(k_R\sqrt{2J_x\left[\beta_{x1}R_{51}^2 - 2\alpha_{x1}R_{51}R_{52} + \gamma_{x1}R_{52}^2\right]}\right) J_0\left(k_R F\sqrt{2J_z\gamma_{z1}}\right),$$

$$\alpha_{III} = \frac{A}{\sqrt{2J_z\gamma_{z1}}} J_0\left(k_R\sqrt{2J_x\left[\beta_{x1}R_{51}^2 - 2\alpha_{x1}R_{51}R_{52} + \gamma_{x1}R_{52}^2\right]}\right) J_1\left(k_R F\sqrt{2J_z\gamma_{z1}}\right), \tag{38}$$

where the horizontal and longitudinal action of a particle in a plannar uncoupled storage ring are defined as

$$J_x \equiv \frac{(x - D_x\delta)^2 + \left[\alpha_x(x - D_x\delta) + \beta_x(x' - D'_x\delta)\right]^2}{2\beta_x},$$

$$J_z \equiv \frac{\left(z - D'_x x - D_x x'\right)^2 + \left[\alpha_z(z - D'_x x - D_x x') + \beta_z\delta\right]^2}{2\beta_z}. \tag{39}$$

In the above equation, we can also write

$$R_{51}D_{x1} + R_{52}D'_{x1} = -\sqrt{\mathcal{H}_{x1}\mathcal{H}_{x2}}\sin(\Delta\psi_{x21} - \Delta\chi_{x21}),$$

$$\beta_{x1}R_{51}^2 - 2\alpha_{x1}R_{51}R_{52} + \gamma_{x1}R_{52}^2 = \mathcal{H}_{x1} + \mathcal{H}_{x2} - 2\sqrt{\mathcal{H}_{x1}\mathcal{H}_{x2}}\cos\left(\Delta\psi_{x21} - \Delta\chi_{x21}\right). \tag{40}$$

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