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WHY IS THE COHERENT RADIATION FROM LASER-INDUCED MICROBUNCHES NARROWBANDED AND COLLIMATED

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Abstract

There are two reasons: one is the long coherence length of radiation from microbunches imprinted by the modulation laser, the second is the finite transverse electron beam size. In other words, one is due to the longitudinal form factor, and the other the transverse form factor of the electron beam. Here we study the role of these form factors in shaping the energy spectrum and spatial distribution of microbunching radiation. The investigations are of value for cases like steady-state microbunching (SSMB), coherent harmonic generation (CHG) and free-electron laser (FEL).

GENERAL FORMULATION

For simplicity, as the first step we consider only the impacts of particle position x, y and z, but ignore the particle angular divergence x', y' and energy deviation δ, on the radiation. We will discuss the requirement of applying this approximation and the impact of beam divergence and energy spread in the end of this paper. With this simplification, the spectrum of radiation from an electron beam with N_e electrons is related to that of a single electron according to

$$\left.\frac{d^2W}{d\omega d\Omega}(\theta,\varphi,\omega)\right|_{\text{beam}} = N_e^2|b(\theta,\varphi,\omega)|^2\left.\frac{d^2W}{d\omega d\Omega}(\theta,\varphi,\omega)\right|_{\text{point}}, \tag{1}$$

with θ and φ being the polar and azimuthal angles in a spherical coordinate system, respectively, and

$$b(\theta,\varphi,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(x,y,z) \, e^{-i\omega\left(\frac{x\sin\theta\cos\varphi+y\sin\theta\sin\varphi}{c}+\frac{z}{\beta c}\right)}dxdydz, \tag{2}$$

in which β is the particle velocity normalized by the speed of light in vacuum c and for relativistic beam can be approximated as 1, and $\rho(x,y,z)$ is the normalized charge density satisfying $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(x,y,z)dxdydz = 1$. When the longitudinal and transverse dimensions of the electron beam are decoupled, we can factorize $b(\theta,\varphi,\omega)$ as

$$b(\theta,\varphi,\omega) = b_\perp(\theta,\varphi,\omega)\times b_z(\omega), \tag{3}$$

where

$$b_\perp(\theta,\varphi,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(x,y)e^{-i\omega\left(\frac{x\sin\theta\cos\varphi+y\sin\theta\sin\varphi}{c}\right)}dxdy, \tag{4}$$

and

$$b_z(\omega) = \int_{-\infty}^{\infty}\rho(z)e^{-i\omega\frac{z}{\beta c}}dz. \tag{5}$$

Note that $\rho(x,y)$ and $\rho(z)$ are then the projected charge density. $b_z(\omega)$ is the usual bunching factor found in literature and is independent of the observation angle. This however is not true for $b_\perp(\theta,\varphi,\omega)$. For example, for an x-y decoupled transversely Gaussian beam, we have

$$|b_\perp(\theta,\varphi,\omega)|^2 = \exp\left\{-\left(\frac{\omega}{c}\right)^2\left[(\sigma_x\sin\theta\cos\varphi)^2+(\sigma_y\sin\theta\sin\varphi)^2\right]\right\}, \tag{6}$$

where $\sigma_{x,y}$ are the RMS beam size in the horizontal, and vertical dimension, respectively.

In order to efficiently quantify the impact of the transverse and longitudinal distributions of an electron beam on the overall radiation energy spectrum, here we define the transverse and longitudinal form factors of an electron beam as

$$FF_\perp(\omega) = \frac{\int_0^\pi\sin\theta d\theta\int_0^{2\pi}d\varphi|b_\perp(\theta,\varphi,\omega)|^2\left.\frac{d^2W}{d\omega d\Omega}(\theta,\varphi,\omega)\right|_{\text{point}}}{\int_0^\pi\sin\theta d\theta\int_0^{2\pi}d\varphi\left.\frac{d^2W}{d\omega d\Omega}(\theta,\varphi,\omega)\right|_{\text{point}}}, \tag{7}$$

and

$$FF_z(\omega) = |b_z(\omega)|^2, \tag{8}$$

respectively. The overall form factor is then

$$FF(\omega) = FF_\perp(\omega)FF_z(\omega). \tag{9}$$

The total radiation energy spectrum of an electron beam is related to that of a single electron by

$$\left.\frac{dW}{d\omega}\right|_{\text{beam}} = N_e^2FF(\omega)\left.\frac{dW}{d\omega}\right|_{\text{point}}. \tag{10}$$

LONGITUDINAL FORM FACTOR

Cleanly Separated Microbunch Train

When there are multiple microbunches cleanly separated from each other with a distance of the modulation laser wavelength λ_L, like that in some of the SSMB scenarios [1–3], the longitudinal form factor is that of the single microbunch multiplied by a macro form factor,

$$FF_{z\text{MB}}(\omega) = FF_{z\text{SB}}(\omega)\left(\frac{\sin\left(N_b\frac{\omega}{c}\frac{\lambda_L}{2}\right)}{N_b\sin\left(\frac{\omega}{c}\frac{\lambda_L}{2}\right)}\right)^2, \tag{11}$$

where the subscripts $_{\text{MB}}$ and $_{\text{SB}}$ mean multi bunch and single bunch, respectively, and N_b is the number of

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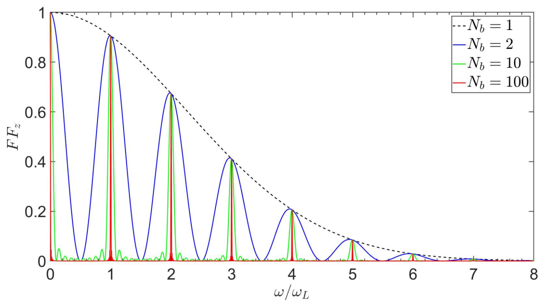

Figure 1: Example plot of the longitudinal form factor for multiple microbunches separated with a distance of laser wavelength λ_L. $\sigma_z = \frac{\lambda_L}{20}$ is used in the calculation.

microbunches. For a Gaussian microbunch we have $FF_{z\mathrm{SB}}(\omega) = \exp\left[-\left(\frac{\omega}{c}\sigma_z\right)^2\right]$, where σ_z is the RMS bunch length. The macro form factor of multi bunches is a periodic function of the radiation frequency, with a period of the modulation laser frequency. The full width at half maximum (FWHM) linewidth around each laser harmonics is

$$\Delta\omega_{\mathrm{FWHM}} = \frac{\omega_L}{N_b}. \quad (12)$$

When N_b goes to infinity, this macro form factor will become the periodic delta function. Note that when we use the above form factor $FF_{z\mathrm{MB}}(\omega)$ to calculate the radiation energy or spectrum from N_b microbunches, the number of electrons used should be $N_b N_{e\mathrm{SB}}$, with $N_{e\mathrm{SB}}$ the number of electrons per microbunch. Figure 1 presents an example plot of the longitudinal form factor of multiple cleanly separatedly microbunches.

Continuous Bunch-based Microbunching

The longitudinal form factor for a coasting or infinite-long beam-based laser-induced microbunching is

$$FF_{z,\mathrm{coasting}}(\omega) = \left(\sum_{n=0}^{\infty}\delta\left(\frac{\omega}{c} - nk_L\right)\right)\left|J_n\left[-\frac{\omega}{c}R_{56}A\right]\right|^2 \exp\left[-\left(\frac{\omega}{c}R_{56}\sigma_\delta\right)^2\right], \quad (13)$$

where $k_L = \frac{2\pi}{\lambda_L}$ is the modulation laser wavenumber, J_n is the n-th order Bessel function of the first kind and

$$\delta(x) = \begin{cases} 1, & x = 0, \\ 0, & \text{else}. \end{cases} \quad (14)$$

Namely there is only bunching at the laser harmonics. For the more-often confronted case of a finite bunch length, like that in CHG and laser-seeded FEL, according to the convolution theorem, then each delta function line in the longitudinal form factor spectrum is broadened by

$$(\Delta\omega)_{\mathrm{FWHM}} = 2\sqrt{2\ln 2}\frac{c/\sigma_z}{\sqrt{2}} = \frac{4\sqrt{2}\ln 2}{(\Delta t)_{\mathrm{FWHM}}}, \quad (15)$$

where we have assumed the bunch before microbuching is Gaussian with an RMS length of σ_z, and Δt is the electron

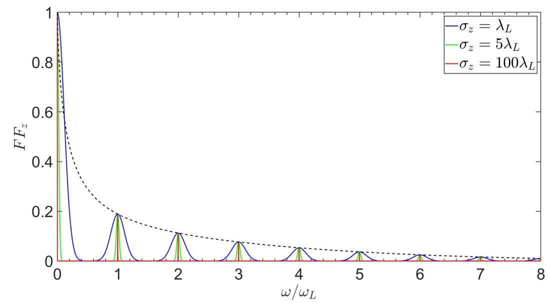

Figure 2: Example plot of the longitudinal form factor of a continuous bunch-based microbunching. The dashed line is $FF_z(\omega) = |J_n\left[-\frac{\omega}{c}R_{56}A\right]|^2 \exp\left[-\left(\frac{\omega}{c}R_{56}\sigma_\delta\right)^2\right]$.

bunch length in unit of time. An example plot of the longitudinal form factor of a continuous bunch-based laser-induced microbunching is given in Fig. 2. As can be seen, the longer the bunch, the narrower the bandwidth around each laser harmonic lines.

Radiation Spatial Distribution and Opening Angle

We use undulator radiation as an example. When the fundamental resonant radiation wavelength is a high harmonic of the modulation laser wavelength, $\lambda_0 = \frac{1+\frac{K^2}{2}}{2\gamma^2}\lambda_u = \frac{\lambda_L}{P}$ where K is the dimensionless planar undulator parameter, γ is the Lorentz factor, λ_u is the period of radiator undulator and P is an integer, corresponding to the peaks in the longitudinal form factor whose frequencies are lower than the on-axis radiation frequency, there will be interference rings in the spatial distribution of the coherent radiation from different microbunches. These rings corresponds to the redshifted undulator radiation, whose polar angles are determined by the off-axis resonant condition,

$$\frac{1+\frac{K^2}{2}+\gamma^2\theta^2}{2\gamma^2}\lambda_u = \frac{\lambda_L}{Q}, \ 1 \le Q < P, \quad (16)$$

where Q is an integer.

For the opening angle of the coherent radiation due to longitudinal form factor, we focus on the case where the relative bandwidth of form factor around the H-th harmonic of on-axis resonant frequency is much narrower than that of the incoherent radiation, $\left(\frac{\Delta\omega}{H\omega_0}\right)_{\mathrm{FWHM}} \ll \frac{1}{HN_u}$, where N_u is the number of undulator periods. The opening angle of the H-th harmonic coherent undulator radiation due to longitudinal form factor in this case is

$$\theta_{||} = \frac{\sqrt{1+\frac{K^2}{2}}}{\sqrt{HN_u}\gamma}. \quad (17)$$

TRANSVERSE FORM FACTOR

As can be seen from Eq. (6), when the transverse electron beam size is large, for off-axis radiation, the effective bunch

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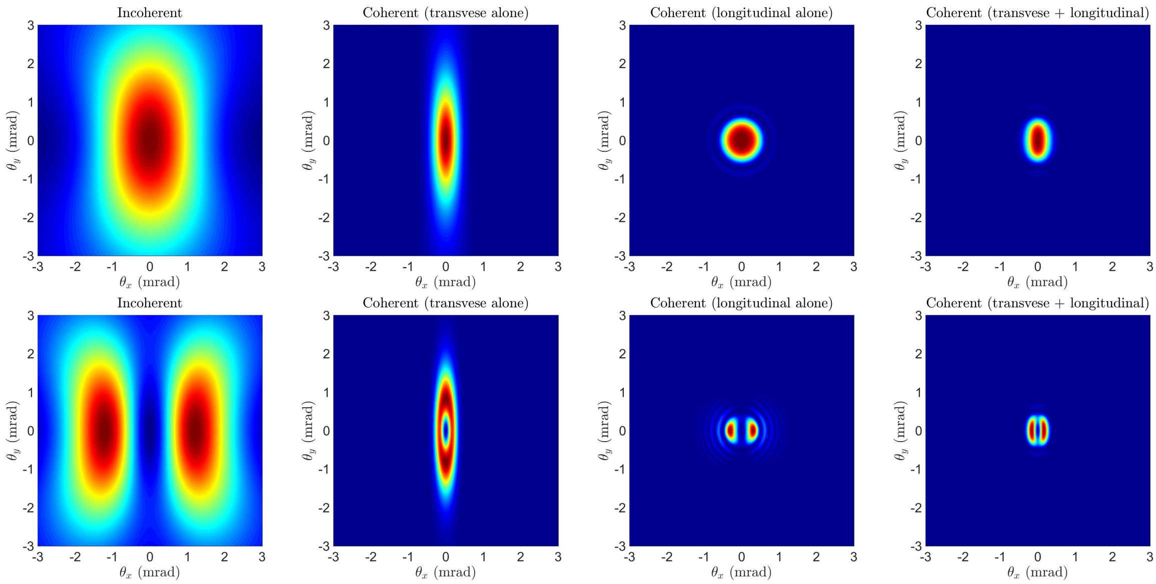

Figure 3: Spatial distribution of the fundamental mode (up) and second harmonic (bottom) coherent undulator radiation energy in SSMB PoP experiment. From left to right: 1) incoherent radiation, 2) coherent radiation but considering only the impact of transverse dimension of electron beam, 3) coherent radiation but considering only the impact of longitudinal dimension of electron beam, 4) coherent radiation considering both the transverse and longitudinal dimensions of electron beam. Parameters used: $E_0 = 250$ MeV, $\lambda_L = \lambda_0 = 1064$ nm, $\lambda_u = 0.125$ m, $N_u = 32$, $\sigma_x = 500\,\mu$m, $\sigma_y = 100\,\mu$m $\sigma_z = 30\,\mu$m (0.1 ps).

lengthening will be significant, thus decreasing the effective bunching factor. Therefore, a large transverse electron beam size will suppress the off-axis redshofted coherent radiation. Therefore, this will also makes the coherent radiation more narrowbanded and collimated in the forward direction, compared to the incoherent radiation.

Since the transverse form factor depends on the specific radiation process involved, there is not an universal formula applies in all cases. For a transversely round Gaussian beam and undulator radiation, we have the approximate transverse form factor for the H-th harmonic undulator radiation [3,4]

$$FF_\perp(H,\omega) = e^{-4N_u\pi S\left(H-\frac{\omega}{\omega_0}\right)} FF_\perp(S), \tag{18}$$

with $FF_\perp(S) = \frac{2}{\pi}\left[\tan^{-1}\left(\frac{1}{2S}\right) + S\ln\left(\frac{(2S)^2}{(2S)^2+1}\right)\right]$, where $S = \frac{\sigma_\perp^2 \frac{\omega}{c}}{L_u}$. The relative bandwidth of H-th harmonic coherent radiation due to transverse form factor is

$$\left.\frac{\Delta\omega_{e^{-1}}}{H\omega_0}\right|_\perp \approx \frac{1}{2H^2\sigma_\perp^2 k_u k_0}. \tag{19}$$

Correspondingly, the opening angle of the H-th harmonic coherent radiation due to the transverse form factor is

$$\theta_\perp \approx \frac{\sqrt{2+K^2}}{2H\gamma\sigma_\perp\sqrt{k_u k_0}}. \tag{20}$$

Summarizing, here we use the SSMB proof-of-principle (PoP) experiment [2,3,5] as an example for calculation to show the impact of transverse and longitudinal form factor on the coherent undulator radiation pattern. As can be seen from Fig. 3, both the longitudinal and transverse distribution of electron beam are of relevance in determining the radiation pattern in SSMB PoP.

IMPACT OF ELECTRON BEAM DIVERGENCE AND ENERGY SPREAD

The approximation of ignoring beam divergence and energy spread of the electron beam applies when the relative change of beam size and bunch length in the radiator is small, i.e., $\beta_{x,y} > L_R$ and $\beta_z > R_{56,R}$. For undulator radiation, that is $\beta_{x,y} > L_u$, $2N_u\lambda_0\sigma_\delta < \sigma_z$, in which σ_z is the length of microbunch.

Now we take into account the impact of beam divergence and energy spread on the coherent radiation in a less rigorous way. As an example, here we assume that the beam is a 6D Gaussian one, and round in the transverse dimension. Further we assume the beam reaches its minimal in all three dimensions at the radiator undulator center, which is desired to get high-power radiation, then the effective transverse and longitudinal form factors are

$$\begin{aligned} FF_\perp(\omega) &= \frac{1}{L_u}\int_{-\frac{L_u}{2}}^{\frac{L_u}{2}} FF_\perp\left(\frac{\left(\sigma_\perp^2 + (\sigma_{\theta_\perp} s)^2\right)\frac{\omega}{c}}{L_u}\right) ds, \\ FF_z(\omega) &= \frac{1}{L_u}\int_{-\frac{L_u}{2}}^{\frac{L_u}{2}} e^{-\left(\frac{\omega}{c}\right)^2\left[\sigma_z^2 + \left(\sigma_\delta \frac{s}{L_u 2N_u\lambda_0}\right)^2\right]} ds \\ &= e^{-\left(\frac{\omega}{c}\right)^2\sigma_z^2}\frac{\sqrt{\pi}}{2}\frac{\operatorname{erf}\left(\frac{\omega}{c}\sigma_\delta N_u\lambda_0\right)}{\frac{\omega}{c}\sigma_\delta N_u\lambda_0}, \end{aligned} \tag{21}$$

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where $\sigma_\perp$, $\sigma_{\theta_\perp}$, σ_z and σ_δ are the transverse beam size, divergence, bunch length and energy spread at the undulator center, with $\sigma_\perp\sigma_{\theta_\perp} = \epsilon_\perp$ and $\sigma_z\sigma_\delta = \epsilon_z$, and $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ is the error function. The key point of considering the impact of beam divergence and energy spread is that there is an optimal choice of beam size and bunch length at the radiator center, given the transverse and longitudinal emittance of an electron beam.

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