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USEFUL FORMULAS AND EXAMPLE PARAMETERS SET FOR THE DESIGN OF SSMB STORAGE RINGS

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Abstract

A promising accelerator light source mechanism called steady-state microbunching (SSMB) has been actively studied in recent years. Here we summarize some important formulas for the design of SSMB storage rings. Generally we group our formulas into two categories, i.e., a longitudinal weak focusing storage ring for a desired radiation wavelength $\lambda_R \gtrsim 100$ nm, and a transverse-longitudinal coupling, or a generalized longitudinal strong focusing, storage ring for a desired radiation wavelength 1 nm $\lesssim \lambda_R \lesssim 100$ nm. In each category, we have presented an example parameters set for the corresponding SSMB storage ring, to generate kW-level infrared, EUV and soft X-ray radiation, respectively.

INTRODUCTION

In Ref. [1], we have conducted indepth theoretical and experimental studies on the steady-state microbunching (SSMB) mechanism [2–18], which promises high-power narrow-band coherent radiation, with wavelength ranging from THz to soft X-ray. To make our investigations more useful for practitioners, especially concerning the parameters choice for an SSMB storage ring, here in this paper we present some important formulas and example parameters set for the design of SSMB storage rings.

LONGITUDINAL WEAK FOCUSING SSMB

The effective modulation voltage of a laser modulator using a planar undulator is [19]

$$V_L = \frac{[JJ]K}{\gamma}\sqrt{\frac{4P_L Z_0 Z_R}{\lambda_L}}\tan^{-1}\left(\frac{L_u}{2Z_R}\right). \quad (1)$$

in which γ is the Lorentz factor, $[JJ] = J_0(\chi) - J_1(\chi)$ and $\chi = \frac{K^2}{4+2K^2}$, J_n is the n-th order Bessel function of the first kind, $K = \frac{eB_0}{m_e c k_u} = 0.934 \cdot B_0[\text{T}] \cdot \lambda_u[\text{cm}]$ is the undulator parameter, determined by the peak magnetic flux density B_0 and period of the undulator λ_u, c the speed of light in free space, P_L is the modulation laser power, $Z_0 = 376.73\ \Omega$ is the impedance of free space, Z_R is the Rayleigh length of the laser, L_u is the undulator length. The linear energy chirp strength around zero-crossing phase is related to the laser and modulator undulator parameters according to $h = \frac{eV_L}{E_0}k_L$, with E_0 the particle energy, $k_L = 2\pi/\lambda_L$ the modulation laser wavenumber.

Linear stability of the longitudinal motion requires $0 < h\eta C_0 < 4$, where C_0 is the ring circumference, η is the phase slippage factor of the ring. In a longitudinal weak focusing ring ($\nu_s \ll 1$), the longitudinal beta function at the laser modulator is

$$\beta_{z\text{S}} \approx \sqrt{\frac{\eta C_0}{h}}. \quad (2)$$

The synchrotron tune is $\nu_s \approx \frac{\eta}{|\eta|}\frac{\sqrt{h\eta C_0}}{2\pi}$. The natural bunch length at the laser modulator is $\sigma_{z\text{S}} = \sigma_{\delta\text{S}}\beta_{z\text{S}}$, where $\sigma_{\delta\text{S}} = \sqrt{\frac{C_q}{J_s}\frac{\gamma^2}{\rho}}$ is the natural energy spread, with ρ the bending radius of dipole in the ring, $C_q = \frac{55\lambda\!\!\!^{-}_e}{32\sqrt{3}} = 3.8319\times10^{-13}$ m, $\lambda\!\!\!^{-}_e = \frac{\lambda_e}{2\pi} = 386$ fm is the reduced Compton wavelength of electron, J_s is the longitudinal damping partition number. The micro-bucket half-height is $\hat{\delta}_{\frac{1}{2}} = \frac{2}{\beta_{z\text{S}}k_L}$.

If there is a single laser modulator in the ring, and if longitudinal damping partition $J_s = 2$, then the theoretical minimum bunch length and longitudinal emittance in a longitudinal weak focusing ring with respect to the bending radius ρ and angle θ of each bending magnet are

$$\begin{aligned}\sigma_{z,\min}[\mu\text{m}] &\approx 4.93\rho^{\frac{1}{2}}[\text{m}]E_0[\text{GeV}]\theta^3[\text{rad}],\\ \epsilon_{z,\min}[\text{nm}] &\approx 8.44E_0^2[\text{GeV}]\theta^3[\text{rad}].\end{aligned} \quad (3)$$

Coherent undulator radiation power at the odd-H-th harmonic from a transversely-round electron beam is

$$P_{H,\text{peak}}[\text{kW}] = 1.183N_u H\chi[JJ]_H^2 FF_\perp(S)|b_{z,H}|^2 I_P^2[\text{A}], \quad (4)$$

where N_u is the number of undulator periods, $[JJ]_H^2 = \left[J_{\frac{H-1}{2}}(H\chi) - J_{\frac{H+1}{2}}(H\chi)\right]^2$, with $\chi = \frac{K^2}{4+2K^2}$, and the transverse form factor $FF_\perp(S) = \frac{2}{\pi}\left[\tan^{-1}\left(\frac{1}{2S}\right) + S\ln\left(\frac{(2S)^2}{(2S)^2+1}\right)\right]$, with $S = \frac{\sigma_\perp^2 \frac{\omega}{c}}{L_u}$ and $\sigma_\perp$ the RMS transverse electron beam size, $b_{z,H}$ is the bunching factor at the H-th harmonic, and I_P is the peak current.

Based on the above formulas, here we present an example parameters set in Tab. 1 of a longitudinal weak focusing SSMB storage ring, aimed for high-power infrared radiation generation. As can be seen, such a compact SSMB storage ring can be used for high-power infrared radiation generation. The requirement on the stored laser power is easy to realize in practice. All the other parameters listed are also within practical range. A sharp reader may notice that the microbucket half-height is only twice the natural energy spread of the electron beam. Therefore, in addition to the shallow microbuckets, we need a larger bucket, for example a barrier bucket formed by an induction linac, to

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Table 1: Example parameters set of a longitudinal weak focusing SSMB storage ring for infrared radiation generation.

Parameter	Value	Description
E_0	250 MeV	Beam energy
C_0	50 m	Circumeference
η	4×10^{-6}	Phase slippage factor
ρ_{ring}	0.6 m	Bending radius of dipoles in the ring
θ	$\frac{\pi}{7}$	Bending angle of each dipole
$\sigma_{\delta S}$	2.76×10^{-4}	Natural energy spread
$\sigma_{z,\text{lim}}$	86 nm	Theoretical lower bunch length limit
λ_L	1064 nm	Modulation laser wavelength
h	500 m^{-1}	Energy chirp strength
σ_{zS}	175 nm	Natural bunch length
$\delta_{\frac{1}{2}}$	5.36×10^{-4}	Microbuket half-height
$\lambda_{u\text{Mod}}$	5 cm	Modulator undulator period
$B_{0\text{Mod}}$	0.92 T	Modulator peak magnetic field
$L_{u\text{Mod}}$	1 m	Modulator undulator length
$P_L(Z_R=\frac{L_u}{3})$	24 kW	Modulation laser power
$\lambda_R=\lambda_L$	1064 nm	Radiation wavelength
b_1	0.59	Bunching factor
$\sigma_\perp$	100 µm	Transverse electron beam size at the radiator
$\lambda_{u\text{Rad}}$	5 cm	Radiator undulator period
$B_{0\text{Rad}}$	0.92 T	Radiator peak magnetic field
$L_{u\text{Rad}}$	2 m	Radiator length
P_R	1 kW @ $I_P=0.55$ A	Radiation peak/average power

constrain the particles in the ring to ensure a large enough beam lifetime.

TRANSVERSE-LONGITUDINAL COUPLING SSMB

Transverse-longitudinal coupling (TLC) dynamics can be invoked to compress the bunch length with a shallow energy modulation strength, by taking advantage of the ultrasmall vertical emittance in a planar ring. For a TLC dynamics-based SSMB, or a generalized longitudinal strong focusing SSMB [15], using TEM00 mode laser modulator for energy modulation, we have the following important formulas.

Relation between energy chirp strength and optical functions at the modulator and radiator

$$h^2(\text{Mod})\mathcal{H}_y(\text{Mod})\mathcal{H}_y(\text{Rad})\geq 1, \tag{5}$$

where $\mathcal{H}_y$ is a chromatic function quantifying the contribution of vertical emittance to bunch length. Bunching factor at the n-th laser harmonic in TLC SSMB at the radiator

$$b_n=\left(\sum_{m=-\infty}^{\infty}J_m(n)\exp\left[-\left((n-m)k_L\sigma_z(\text{Mod})\right)^2/2\right]\right)\exp\left[-\left(nk_L\sigma_z(\text{Rad})\right)^2/2\right], \tag{6}$$

where $\sigma_z(\text{Mod}) = \sqrt{\epsilon_z\beta_z(\text{Mod})+\epsilon_y\mathcal{H}_y(\text{Mod})}$ and $\sigma_z(\text{Rad})=\sqrt{\epsilon_y\mathcal{H}_y(\text{Rad})}$ are the linear bunch length at the modulator and radiator, respectively.

Contribution of two modulators to ϵ_y from quantum excitation

$$\Delta\epsilon_y(\text{Mod})=2\times\frac{55}{96\sqrt{3}}\frac{\alpha_F\lambda_e^2\gamma^5}{\alpha_V}\frac{\mathcal{H}_y(\text{Mod})}{\rho_{0\text{Mod}}^3}\frac{4}{3\pi}L_u, \tag{7}$$

where $\alpha_F=\frac{1}{137}$ is the fine-structure constant, $\alpha_V\approx\frac{U_0}{2E_0}$ is the vertical damping rate with U_0 the radiation energy loss of a particle per turn. Assuming $\epsilon_y=\Delta\epsilon_y(\text{Mod})$, which means the vertical emittance is solely from the two modulators, then the required modulation laser power and modulator length scaling are

$$\begin{aligned}P_L[\text{kW}]&\approx 5.67\frac{\lambda_L^{\frac{7}{3}}[\text{nm}]E_0^{\frac{8}{3}}[\text{GeV}]B_{0\text{Mod}}^{\frac{7}{3}}[\text{T}]}{\sigma_z^2(\text{Rad})[\text{nm}]B_{\text{ring}}[\text{T}]},\\ L_u[\text{m}]&\approx 57\frac{B_{\text{ring}}[\text{T}]\epsilon_y[\text{pm}]}{\mathcal{H}_y(\text{Mod})[\mu\text{m}]B_{0\text{Mod}}^3[\text{T}]},\end{aligned} \tag{8}$$

where $B_{0\text{Mod}}$ is the peak magnetic flux density of the modulator undulator, B_{ring} is the magnetic flux density of bending magnets in the ring. The above scaling laws are accurate when $K_u>\sqrt{2}$. For the more general case, refer to Ref. [1].

Based on the presented formulas, here we present an example parameters set in Tab. 2 of a TLC SSMB storage ring, aimed for high-power EUV and soft X-ray radiation. Since the bunch lengthening from vertical emittance at the modulator will be comparable or longer than the modulation laser wavelength, the difference of final bunching factor at the radiator between a pre-microbunched beam and coasting beam is negligible. To minimize the IBS effect, we choose to use

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Table 2: Example parameters set of a TLC SSMB storage ring for EUV and soft X-ray radiation generation.

	Parameter	Value	Description
	E_0	800 MeV	Beam energy
	C_0	$\lesssim$ 100 m	Circumference
	ρ_{ring}	2 m	Bending radius in the ring
	$\sigma_{\delta S}$	4.85×10^{-4}	Natural energy spread
	ϵ_y	2 pm	Vertical emittance
	λ_L	270 nm	Modulation laser wavelength
	$\sigma_\perp$	10 µm	Effective transverse electron beam size at the radiator
EUV (13.5 nm)	σ_z(Rad)	2 nm	Linear bunch length at the radiator
	σ_{zy}(Mod)	1.85 μm	Bunch lengthening from ϵ_y at the modulator
	h	541 m^{-1}	Energy chirp strength
	$\lambda_{u\text{Mod}}$	0.5 m	Modulator undulator period
	$B_{0\text{Mod}}$	0.039 T	Modulator peak magnetic flux density
	$L_{u\text{Mod}}$	1.5 m	Modulator length
	$P_L(Z_R = \frac{L_u}{3})$	141 kW	Modulation laser power
	$\lambda_R = \frac{\lambda_L}{20}$	13.5 nm	Radiation wavelength
	b_{20}	0.11	Bunching factor
	$\lambda_{u\text{Rad}}$	2 cm	Radiator undulator period
	$B_{0\text{Rad}}$	1.15 T	Radiator peak magnetic flux density
	$L_{u\text{Rad}}$	3.2 m	Radiator length
	P_R	1 kW @ $I_P = 1.5$ A	Radiation peak/average power
Soft X-ray (6.75 nm)	σ_z(Rad)	1 nm	Linear bunch length at the radiator
	σ_{zy}(Mod)	1.85 µm	Bunch lengthening from ϵ_y at the modulator
	h	1082 m^{-1}	Energy chirp strength
	$\lambda_{u\text{Mod}}$	0.5 m	Modulator undulator period
	$B_{0\text{Mod}}$	0.039 T	Modulator peak magnetic flux density
	$L_{u\text{Mod}}$	1.5 m	Modulator length
	$P_L(Z_R = \frac{L_u}{3})$	564 kW	Modulation laser power
	$\lambda_R = \frac{\lambda_L}{40}$	6.75 nm	Radiation wavelength
	b_{40}	0.085	Bunching factor
	$\lambda_{u\text{Rad}}$	1.5 cm	Radiator undulator period
	$B_{0\text{Rad}}$	1.11 T	Radiator peak magnetic flux density
	$L_{u\text{Rad}}$	2.4 m	Radiator length
	P_R	1 kW @ $I_P = 2.2$ A	Radiation peak/average power
Soft X-ray (2.7 nm)	σ_z(Rad)	0.5 nm	Linear bunch length at the radiator
	σ_{zy}(Mod)	2.81 µm	Bunch lengthening from ϵ_y at the modulator
	h	1423 m^{-1}	Energy chirp strength
	$\lambda_{u\text{Mod}}$	0.6 m	Modulator undulator period
	$B_{0\text{Mod}}$	0.028 T	Modulator peak magnetic flux density
	$L_{u\text{Mod}}$	1.8 m	Modulator length
	$P_L(Z_R = \frac{L_u}{3})$	1 MW	Modulation laser power
	$\lambda_R = \frac{\lambda_L}{100}$	2.7 nm	Radiation wavelength
	b_{100}	0.049	Bunching factor
	$\lambda_{u\text{Rad}}$	1 cm	Radiator undulator period
	$B_{0\text{Rad}}$	0.86 T	Radiator peak magnetic flux density
	$L_{u\text{Rad}}$	2 m	Radiator length
	P_R	1 kW @ $I_P = 5$ A	Radiation peak/average power

a coasting beam for microbunching here. It can be seen that as long as we can realize a coasting beam of 1.5 A average current, and an optical cavity stored power of $\gtrsim$ 100 kW, we can realize 1 kW average power 13.5 nm EUV. Even if we can only realize an average beam current of 1 A or less, we can take advantage of the fact that $P_{\text{coh}} \propto I_P^2$ to realize an average radiation power of kW level, by decreasing the filling factor of electron beam in the ring but increasing the peak current as long as the value is below the collective instability threshold. Since there is no requirement on the longitudinal emittance for a coasting beam, the ring can be very compact, for example a circumference of 100 m or even smaller should

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be feasible. This compact high-power EUV radiation source is promising to fulfill the urgent need of EUV lithography for high volume manufacture of computer chips, and also serve the future lithography like Blue-X which invokes 6.x nm-wavelength light source. Such an SSMB-based high-power high-flux soft X-ray photon source could be of great value for fundamental science like high-resolution angle-resolved photoemission spectroscopy and can also bridge the water window gap.

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