MINIMIZING THE FLUCTUATION OF RESONANCE DRIVING TERMS FOR ANALYZING AND OPTIMIZING THE STORAGE RING DYNAMIC APERTURE

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Abstract

Minimization of resonance driving terms (RDTs) of nonlinear magnets such as sextupoles and octupoles is an essential condition for enlarging the dynamic aperture (DA) of a storage ring. We recently studied the correlation between minimizing the fluctuation or variation of RDTs along the ring and enlarging the DA. It was found that minimizing the RDT fluctuations is much more effective than minimizing the commonly-used one-turn RDTs in enlarging the DA, and that reducing low-order RDT fluctuations can also help reduce both higher-order RDT fluctuations and higher-order one-turn RDTs. In this paper, the DA analysis based on minimizing RDT fluctuations is further extended. By considering the RDT fluctuations including low- and high-frequency fluctuations, some nonlinear dynamics issues can be explained. The DA optimization is also studied based on numerically minimizing RDT fluctuations using genetic algorithms. Large DAs can be obtained, and the optimization is performed very fast.

INTRODUCTION

Enlarging the dynamic aperture (DA) of a storage ring is critical for improving beam injection efficiency and beam lifetime. At present, the numerical approach based on particle tracking and evolutionary algorithms has been intensively developed and successfully applied to the DA optimization of many storage ring light sources, which can in principle obtain the global best solution. But in this approach, it is hard to provide the physics for further nonlinear analysis and feedback on linear optics adjustment. Minimization of resonance driving terms (RDTs) [1] is a widely-used analytical approach, which has been used for nonlinear analysis and optimization for decades. In this traditional analytical approach, controlling RDTs and amplitude dependent tune shifts (ADTS) can give a larger DA, but it is only a necessary condition. Just recently, we found that minimizing the fluctuation or variation of RDTs along the ring is much more effective than minimizing the commonly-used one-turn RDTs in enlarging the DA [2], which can enhance the capability of this analytical approach.

In this paper, we will first briefly describe the correlation between minimizing RDT fluctuations and enlarging DA, and the physics behind this correlation. Then taking the SOLEIL II storage ring lattice as an example, this paper studies the DA analysis with low- and high-frequency RDT fluctuations, and the DA optimization based on minimizing RDT fluctuations.

ANALYZING DA BASED ON MINIMIZ-ING RDT FLUCTUATIONS

Correlation Between RDT Fluctuations and DA

For the hybrid multi-bend achromat (MBA) lattice and higher-order-achromat based MBA lattice, which have been adopted in many designs of 4th-generation synchrotron light sources, main nonlinear effects caused by sextupoles can be cancelled within a lattice cell. This kind of nonlinear cancellation is more effective than the nonlinear cancellation over some lattice cells, which was used in designing some 3rd-generation light sources. That is why the DAs of some 4th-generation light source designs can allow off-axis injection, even though their emittances are 1~2 orders of magnitude lower than those of 3rd-generation light sources. For the nonlinear cancellation within a lattice cell, the variation of RDTs along the longitudinal position is smaller than the case of the nonlinear cancellation over some lattice cells. This inspires us that reducing the variation or fluctuation of RDTs could be very beneficial for enlarging the DA.

Recently, we studied the correlation between minimizing RDT fluctuations and enlarging DA with different lattices [2]. In our study, the fluctuation of a RDT is quantitatively represented by the average RDT at the positions of sextupoles and octupoles. The results of DA analysis showed that minimizing RDT fluctuations is much more effective than minimizing one-turn RDTs. In other words, a nonlinear solution with smaller RDT fluctuations has a much higher probability of having a larger DA than a solution with smaller one-turn RDTs. In addition, by calculating RDT fluctuations, we can analyze which RDTs are likely to have a relatively large effect on the DA.

Physics Behind Minimizing RDT Fluctuations

We also studied the correlation between low-order RDTs and higher-order RDTs. It was found that reducing low-order RDT fluctuations can also help reduce both higher-order RDT fluctuations and higher-order one-turn RDTs [2]. By this logic, if higher-order RDT fluctuations are reduced, even higher-order RDTs can also be reduced. By using frequency map analysis, it was demonstrated that a 5th-order RDT of the SSRF lattice can be controlled if 3rd-order RDT fluctuations are reduced [2]. Besides, we also found that reducing 3rd-order RDT fluctuations can also help reduce

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ADTS terms [3]. While for the commonly-used one-turn RDTs, even if the 3rd-order one-turn RDTs are smaller, the 4th-order one-turn RDTs can be larger. Figure 1 shows the schematic of the correlation between low- and higher-order RDTs, which is also the physics behind the strong correlation between minimizing RDT fluctuations and enlarging DA.



Figure 1: Schematic of the correlation between low- and higher-order RDTs.

Since reducing low-order RDT fluctuations can also reduce higher-order and even higher-order RDTs, we can mainly consider low-order RDT fluctuations if we analyze and optimize DA based on minimizing RDT fluctuations, and it would not be necessary to calculate higher-order RDTs. As we know, for RDTs higher than 4th-order, they are not only more computationally complicated but also more numerous. Note that octupoles are used in many 4thgeneration light source designs, so we need to calculate the 4th-order RDT fluctuations for such designs.

Low- and High-frequency RDT Fluctuations

The RDT fluctuations are like waves, and they can be considered to include low- and high-frequency fluctuations. The low-frequency RDT fluctuations are related to the nonlinear cancellation over some lattice cells, and the high-frequency fluctuations are related to the cancellation within a lattice cell. Based on a simplified model, we can use lowand high-frequency RDT fluctuations to analyze some nonlinear dynamics.

Here we take the CDR lattice of SOLEIL II [4] as an example, which has two super-periods with the betatron tunes of 54.2/18.2 (H/V). If the tunes are chosen to be 53.2/17.2, which are close to odd numbers, there will be a rough -I transformation between the two super-periods, which is beneficial for sextupole cancellation and thus the increase in DA. Furthermore, the tunes 53.2/17.2 are smaller than 54.2/18.2, which is also beneficial for nonlinear dynamics. So it is likely that the CDR lattice with the tunes of 53.2/17.2 can have a larger DA. However, the tracking-based optimization showed an opposite result.

The low-frequency RDT fluctuations can be used to explain the above problem. As shown in Fig. 2, the nonlinear lattice of the whole ring can basically be seen as consisting of mainly four identical parts, and these parts can be further modelled as "macro-particles" located at the middle positions of these parts. A macro-particle represents all nonlinear magnets of a part. In Figure 2, the betatron tunes between these macro-particles are also given. To calculate RDTs, each macro-particle was treated as a thin sextupole, and the values of sextupole strength and beta functions at the positions of macro-particles were all set to 1. The calculated low-frequency RDT fluctuations are shown in Figs. 3 and 4 for the tunes of 54.2/18.2 and 53.2/17.2, respectively. We see that overall the lattice with the tunes of 53.2/17.2 has larger RDT fluctuations, and this is probably why this lattice has a smaller DA in the numerical optimization.



Figure 2: Simplified model for calculating low-frequency RDT fluctuations of the SOLEIL II CDR lattice with beta-tron tunes of 54.2/18.2 and 53.2/17.2.



Figure 3: Low-frequency fluctuations of 3rd- and 4th-order RDTs for the betatron tunes of 54.2/18.2.



Figure 4: Low-frequency fluctuations of 3rd- and 4th-order RDTs for the betatron tunes of 53.2/17.2.

For the SOLEIL II CDR lattice, it was also found that if the horizontal beta function at the injection straight section and its symmetric straight is increased to a larger value of

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about 20 m with the betatron tunes of 54.2/18.2 unchanged, surprisingly the DA is not increased in the numerical optimization. By using low-frequency RDT fluctuations, we also explained the possible reason based on the same simplified model. Note that the tunes between macro-particles are slightly changed when the horizontal beta function is increased. Here we will not show the results of low-frequency RDT fluctuations for this problem.

OPTIMIZING DA BASED ON MINIMIZ-ING RDT FLUCTUATIONS

For the SOLEIL II TDR lattice [5], we preliminarily optimized its DA based on numerically minimizing RDT fluctuations using genetic algorithms. The result showed that large DAs can be obtained, and the optimization was performed very fast as compared to the tracking-based numerical approach. The RDT fluctuations and DA of the reference solution are shown in Figs. 5 and 6, respectively; and Figs. 7 and 8 show the corresponding results of one optimized solution. The reference solution is from Ref. [5]. It can be seen that the optimized solution has smaller RDT fluctuations and its horizontal DA is larger in the negative direction, which is also the direction of beam injection.







Figure 6: DA of the reference solution of the SOLEIL II TDR lattice, with colour bar indicating diffusion rate.



Figure 7: Fluctuation of 3rd- and 4th-order RDTs of one optimized solution.



In the nonlinear optimization, the tune shifts with momentum also need to be controlled. Based on minimizing RDT fluctuations, the nonlinear optimization of the SO-LEIL II lattice, including both DA optimization and control of tune shifts with momentum, is ongoing. Of course, in the vicinity of the optimized solutions, we can further use tracking-based numerical optimization to search for better solutions.

CONCLUSION

Minimizing the fluctuation or variation of RDTs along the longitudinal position is much more effective than minimizing the commonly-used one-turn RDTs in enlarging the DA. The physical reason is that reducing low-order RDT fluctuations can also help reduce both higher-order RDT fluctuations and higher-order one-turn RDTs. The RDT fluctuations can be considered to have low- and highfrequency fluctuations. Based on a simplified model, the nonlinear dynamics problems of the SOLEIL II lattice can be explained by using low-frequency RDT fluctuations. Based on minimizing RDT fluctuations using genetic algorithms, large DA solutions of the SOLEIL II lattice can be found very fast. Further development for the approach of minimizing RDT fluctuations is ongoing.

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